Topological quantum groups and harmonic analysis

May 15-19, 2017

SNU (Seoul National University), Korea

**Supported by Korea Institute for Advanced Study (KIAS)/
Department of Mathematical Sciences, SNU**

Program

May 15 (Monday)

9:00 – 10:00 **Opening/ Julien Bichon 1 (Universite Blaise Pascal)**

Homological invariants of discrete quantum groups

10:00 – 10:30 **Coffee Break**

10:30 – 11:30 **Paweł Kasprzak (Warsaw University)**

Group-like projections in locally compact quantum groups

11:30 – 12:00 **Yulia Kuznetsova (Universite de Franche-Comte)**

A duality construction for von Neumann bialgebras not involving a Haar weight

12:00 – 2:00 **Lunch Break**

2:00 – 3:00 **Makoto Yamashita 1 (Ochanomizu University)**

Categorical duality for actions of compact quantum groups

3:00 – 3:30 **Paweł Joziak (IMPAN)**

On a quantum Bernstein Theorem

3:30 – 4:00 **Coffee Break**

4:00 – 4:30 **Ami Viselter (University of Haifa)**

Around Property (T) for quantum groups

4:30 – 5:00 **Marco Matassa (Universite Blaise Pascal)**

On the Parthasarathy formula for quantized irreducible flag manifolds

May 16 (Tuesday)

9:00 – 10:00 **Christian Voigt 1 (University of Glasgow)**

Complex semisimple quantum groups and representation theory

10:00 – 10:30 **Coffee Break**

10:30 – 11:30 **Kenny De Commer (Vrije Universiteit Brussel)**

I-factorial quantum torsors and Heisenberg algebras of quantized enveloping type

11:30 – 12:00 **Sang-Gyun Youn (Seoul National University)**

Similarity property of convolution algebras of locally compact quantum groups

12:00 – 12:30 **Nhan-Phu Chung (Sungkyunkwan University)**

Rigidity for group actions

12:30 – 2:00 **Lunch Break**

2:00 – 3:00 **Julien Bichon 2 (Universite Blaise Pascal)**

Homological invariants of discrete quantum groups

3:00 – 3:30 **Martijn Caspers (Utrecht University)**

Weight lifting with ultrapowers

3:30 – 4:00 **Coffee Break**

4:00 – 5:00 **Panel Discussion/Open problems 1**

May 17 (Wednesday)

9:00 – 10:00 **Makoto Yamashita 2 (Ochanomizu University)**

Categorical duality for actions of compact quantum groups

10:00 – 10:30 **Coffee Break**

10:30 – 11:00 **Anna Wysoczanska-Kula (Uniwersytet Wrocławski)**

Lévy-Khintchine type decomposition on universal compact quantum groups

11:00 – 11:30 **J.P. McCarthy (Cork Institute of Technology)**

The Diaconis-Shahshahani Upper Bound Lemma for Finite Quantum Groups

11:30 – 12:00 **Yuki Arano (Kyoto University)**

Compact Lie group actions with continuous Rokhlin property

12:00 – 2:00 **Lunch Break**

2:00 – 3:00 **Christian Voigt 2 (University of Glasgow)**

Complex semisimple quantum groups and representation theory

3:00 – 3:30 **Issan Patri (Chennai Mathematical Institute)**

Topologies on spaces of Von-Neumann algebras and Quantum Groups

3:30 – 4:00 **Coffee Break**

4:00 – 5:00 **Panel Discussion/Open problems 2**

May 18 (Thursday)

9:00 – 10:00 **Julien Bichon 3 (Universite Blaise Pascal)**

Homological invariants of discrete quantum groups

10:00 – 10:30 **Coffee Break**

10:30 – 11:30 **Jason Crann (Carleton University)**

Homological manifestations of quantum group duality

11:30 – 12:00 **Piotr M. Hajac (IMPAN)**

From the noncontractibility of compact quantum groups to a noncommutative Borsuk-Ulam-type conjecture

12:00 – 1:00 **Lunch Break**

1:00 – **Excursion/ Workshop Dinner**

May 19 (Friday)

9:00 – 10:00 **Makoto Yamashita 3 (Ochanomizu University)**

Categorical duality for actions of compact quantum groups

10:00 – 10:30 **Coffee Break**

10:30 – 11:30 **Christian Voigt 3 (University of Glasgow)**

Complex semisimple quantum groups and representation theory

11:30 – 12:00 **Byung-Jay Kahng (Canisius College)**

Invariant weights on a locally compact quantum groupoid

12:00 – 12:30 **Michał Banacki (University of Gdansk)**

On the filtration preserving quantum symmetry groups of non-commutative tori

12:30 – 2:00 **Lunch Break**

2:00 – 3:00 **Mehrdad Kalantar (University of Houston)/Closing**

Open Quantum Subgroups and Induced Representations

3:00 – **Free discussion**

Abstract

- **Yuki Arano (University of Tokyo)**
  Compact Lie group actions with continuous Rokhlin property
  Izumi introduced the Rokhlin property for finite group actions on C*-algebras to classify such actions. In this talk, we give a classification of continuous Rokhlin actions of compact Lie groups with Hodgkin condition on Kirchberg algebras. The proof uses the idea of quantum group theory, especially the dual Baum-Connes for compact groups proven by Meyer and Nest.

- **Michał Banacki (University of Gdansk)**
  On the filtration preserving quantum symmetry groups of noncommutative tori
  We will discuss necessary conditions for a compact quantum group to act on the noncommutative n-torus $\mathbb{T}^n_\theta$ in a filtration preserving way with respect to the given state (in the sense of T. Banica and A. Skalski). As a result, we will construct a family of compact quantum groups $\mathbb{G}^n_\theta = (A^n_\theta, \Delta)$ such that for each θ, $\mathbb{G}^n_\theta$ is the final object in the category of all compact quantum groups acting on $\mathbb{T}^n_\theta$ in a filtration preserving way. We shall describe the structure of the C*-algebra A^n_θ. Finally, we will comment on the representation theory of $\mathbb{G}^n_\theta$. Joint work with M. Marciniak.

- **Julien Bichon (Universite Blaise Pascal)**
  Homological invariants of discrete quantum groups
  This series of lectures will deal with some homological invariants for discrete quantum groups: cohomological dimension and L^2-Betti numbers. After some recollection on discrete and compact quantums and homological algebra, we will outline the construction of these invariants, and will finish with the presentation of some recent computations.

  Plan: (1) Hopf algebras, discrete and compact quantum groups.

  (2) Homological algebra.

  (3) Cohomological dimensions and L^2-Betti numbers of discrete quantum groups.

- **Martijn Caspers (Utrecht University)**
  Weight lifting with ultrapowers
  For a family of von Neumann algebras M_j equipped with normal weights φ_j we define the ultraproduct weight $\varphi = (\varphi_j)_\omega$ on the Raynaud ultrapower $\prod_{j,\omega} M_j$. We study modular theory by considering ultraproducts of Connes' spatial derivatives. This extends earlier results by Ando-Haagerup and Raynaud for the case of bounded functionals, giving yet another proof. We also provide a new proof of a famous result by Raynaud that says that $L^p(\prod_{j,\omega} M_j)$ is isomorphic to $\prod_{j,\omega} L^p(M_j)$.

- **Nhan-Phu Chung (Sungkyunkwan University)**
  Rigidity for group actions
  In this talk, we will present certain rigidity results for group actions on compact spaces. In the first part, we will provide a new characterization of one end groups via cocycle superrigidity of their full shifts. As a consequence, we have an application in continuous orbit equivalence rigidity. In the second part, we prove that if an action of a finitely generated group is expansive and has the pseudo-orbit tracing property then it is C^0 local rigid. A new characterization of subshifts of finite type over finitely generated groups in term of pseudo-orbit tracing property is also mentioned. The first part is joint with Yongle Jiang and the second part is joint work with Keonhee Lee.

- **Kenny De Commer (Vrije Universiteit Brussel)**
  I-factorial quantum torsors and Heisenberg algebras of quantized enveloping type
  A I-factorial quantum torsor consists of an integrable, free and ergodic action of a locally compact quantum group on a type I-factor. We show how such actions admit a nice duality theory. As an example, we consider a deformed Heisenberg algebra associated to a quantum Borel algebra of a semisimple complex Lie algebra g. We show that, endowed with a *-structure swapping the two quantum Borel algebras inside, it allows a completion into a I-factorial quantum torsor for (an amplification of) the von Neumann algebraic completion of the compact form of the quantized enveloping algebra of g.

- **Jason Crann (Carleton University)**
  Homological manifestations of quantum group duality
  We present a homological characterization of quantum group amenability in terms of injectivity of the dual $L^\infty(\hat{G})$ as an operator module over its predual $L^1(\hat{G})$. We then discuss several applications, including a hereditary property of amenability, decomposability of completely bounded $L^1(\hat{G})$-module maps, and a simplified proof of Leptin's theorem for discrete quantum groups. Time permitting, we will present recent work on a notion of inner amenability for quantum groups along with various applications.

- **Piotr M. Hajac (Institute of Mathematics Polish Academy of Sciences)**
  From the noncontractibility of compact quantum groups to a non-commutative Borsuk-Ulam-type conjecture
  The only contractible compact Hausdorff topological group G is the trivial one. This classical fact is easily equivalent to the statement that there exists a continuous equivariant map from the join G*G to G if and only if G is trivial. Remembering that all continuous equivariant maps from a topological group to itself (shift maps) are homeomorphisms, one can see the above equivalence as a special case of the equivalence of a Borsuk-Ulam-type conjecture for free continuous actions of G on a compact Hausdorff space X, and the homotopic nontriviality of equivariant continuous maps from X to X. The aim of this talk is to explain how the above claims of classical topology, with applications ranging from the Brouwer fixed-point theorem to the Hilbert-Smith conjecture, generalize to the realm of compact quantum groups acting freely on unital C*-algebras.

In particular, after translating the Borsuk-Ulam-type conjecture into C*-algebras and extending it to the noncommutative setting, we will prove the noncommutative Borsuk-Ulam-type conjecture for compact quantum groups containing nontrivial finite classical subgroups (torsion), and show some variations of this theorem obtainable through K-theory. Time permitting, we will also discuss very recent results of S.L. Woronowicz and A. Chirvasitu who respectively proved the invertibility of shift maps for arbitrary locally compact quantum groups and classified them in the reduced and full cases. Based on joint work with L. Dabrowski and S. Neshveyev.

- **Paweł Joziak (Institute of Mathematics Polish Academy of Sciences)**
  On a quantum Bernstein Theorem
  The classical Theorem of Bernstein states that a random vector consisting of independent entries with the property that its entries are still independent after applying a generic rotation, is necessarily a Gaussian vector. A similar type of result was obtained by Nica, where "independent" was replaced with "free", and Gauss law was replaced with Wigner law. We pursue a similar type of question with "rotation" replaced with "quantum rotation". Staying in the framework of operator-valued free probability, we show that a random vector with free entries having the property that its entries remain free after applying a quantum family of rotations (described by a quotient of $\mathcal{O}(O_d^+)$) is necessarily a semicircular family of random variables, provided that this quantum family of rotations is not a subset of quantum hyperoctahedral group (the aforementioned quotient of $\mathcal{O}(O_d^+)$ does not factor through $\mathcal{O}(H_d^+)$). We also show that the result is optimal, in the sense that there exist non-semicircular free random variables that remain free after applying the rotations from H_d^+.

- **Byung-Jay Kahng (Canisius College)**
  Invariant weights on a locally compact quantum groupoid
  Motivated by the purely algebraic notion of "weak multiplier Hopf algebras", we develop the definition of a class of locally compact quantum groupoids in the C*-algebra framework. Existence of a certain canonical idempotent element plays an important role. As in the quantum group case, we require left and right Haar weights but the antipode is not explicitly defined. This class would contain all locally compact quantum groups, and form a self-dual category.

  In this talk, we will focus on how to formulate the left and right invariance conditions, similar to but different from the quantum group case. We will gather some alternative forms of the invariant conditions. Then we will explore the central roles these invariant weights play in the quantum groupoid theory, in the construction of the regular representations (in terms of certain partial isometries) and the antipode map.

  (*). This is based on an on-going joint work with Alfons Van Daele (Leuven).

- **Mehrdad Kalantar (University of Houston)**
  Open Quantum Subgroups and Induced Representations
  We introduce the notion of open quantum subgroups of locally compact quantum groups, and then show the two proposed constructions of induced representations, by Kustermans and by Vaes, are both equivalent in the setting of open quantum subgroups to Rieffel's (rather simple) construction of induced representations in the context of C^*-algebras. This talk is based on joint work with Paweł Kasprzak, Adam Skalski, and Piotr Sołtan.

- **Paweł Kasprzak(Warsaw University)**
  Group-like projections in locally compact quantum groups
  In this talk I will present the relation between group-like projections and the concepts of an open quantum subgroup and idempotent state. The role of a group-like projection in the construction of induced representations from an open quantum subgroup will be discussed.I will also give a relatively simple W*-proof of the Salmi's result that characterises compact quantum subgroups of a given quantum groups in terms of normal expected coideals. This talk is based on a few recent papers written with M. Kalantar, F. Khosravi, A. Skalski, P. Sołtan and on ongoing project with R. Faal.

- **Kuznetsova, Yulia (Universite de Franche-Comte)**
  A duality construction for von Neumann bialgebras not involving a Haar weight
  We construct a certain dual algebra for a wide class of operator bialgebras including locally compact quantum groups (LCQG). The dual in this sense of a von Neumann algebraic LCQG A is the enveloping von Neumann algebra $\hat{A}_u^{**}$ of the universal dual $\hat{A}_u$ of Kustermans; but applied to $\hat{A}_u^{**}$, the construction yields A_u^{**}, where A_u is the universal algebra of the original quantum group. In the classical case, it puts in duality the algebras $C_0(G)^{**}$ and $C^*(G)^{**}$ of a locally compact group G. The motivation is, from one side, conceptual: to show that there is no need to know the Haar measure/weight or a multiplicative unitary to construct the dual of a locally compact quantum group, thus contributing to the problem of defining a quantum group in a measure-independent way. From the other side, there is more practical outcome: from given von Neumann bialgebras, even rather badly behaved, the proposed construction yields bialgebras with relatively good properties.

- **Marco Matassa (Universite Blaise Pascal)**
  On the Parthasarathy formula for quantized irreducible flag manifolds
  The Parthasarathy formula expresses the square of the Dirac operator on a symmetric space in terms of central elements of the corresponding enveloping algebra. We investigate whether a result of this type also holds for quantized irreducible flag manifolds, using the Dolbeault-Dirac operators introduced by Kr?hmer and Tucker-Simmons. We show that a Parthasarathy-type formula requires certain quadratic commutation relations in the quantum Clifford algebra defined by the named authors. For quantum projective spaces these relations holds, and we obtain a result which is as close as possible to the

classical case. On the other hand this is not the case for all other irreducible flag manifolds.

- **J.P. McCarthy (Cork Institute of Technology)**
  The Diaconis-Shahshahani Upper Bound Lemma for Finite Quantum Groups
  A central tool in the study of ergodic random walks on finite groups is the Upper Bound Lemma of Diaconis & Shahshahani. The Upper Bound Lemma uses the representation theory of the group to generate upper bounds for the distance to random and thus can be used to determine convergence rates for ergodic walks. These ideas are generalised to the case of finite quantum groups.

- **Issan Patri (Chennai Mathematical Institute)**
  Topologies on spaces of Von-Neumann algebras and Quantum Groups
  In this talk, we will discuss Effros-Marechal and Chistensen topologies on spaces of von-Neumann algebras and discuss properties of the subspaces of certain MASAs and approximation properties. We will also study a topology on spaces of compact quantum groups and connections to the Effros-Marechal topology.

- **Ami Viselter (University of Haifa)**
  Around Property (T) for quantum groups
  Kazhdan's Property (T) is a notion of fundamental importance, with numerous applications in various fields of mathematics such as abstract harmonic analysis, ergodic theory and operator algebras. By using Property (T), Connes was the first to exhibit a rigidity phenomenon of von Neumann algebras. Since then, the various forms of Property (T) have played a central role in operator algebras, and in particular in Popa's deformation/rigidity theory.

  This talk is dedicated to some recent progress in the notion of Property (T) for locally compact quantum groups. Most of our results are concerned with second countable discrete unimodular quantum groups with low duals. In this class of quantum groups, Property (T) is shown to be equivalent to Property $(T)^{1,1}$ of Bekka and Valette. As applications, we extend to this class several known results about countable groups, including theorems on "typical" representations (due to Kerr and Pichot) and on connections of Property (T) with spectral gaps (due to Li and Ng) and with strong ergodicity of weakly mixing actions on a particular von Neumann algebra (due to Connes and Weiss).

  Joint work with Matthew Daws and Adam Skalski.

- **Christian Voigt (University of Glasgow)**
  Complex semisimple quantum groups and representation theory
  Complex quantum groups are certain deformations of classical complex semisimple Lie groups like $SL(n,\mathbb{C})$, obtained using the Drinfeld double construction. We shall first carefully explain the definition of these quantum groups, and then discuss their structure and representation theory. In addition, we will indicate how complex quantum

groups are connected with problems in the study C^*-tensor categories, noncommutative geometry and K-theory.

- **Anna Wysoczańska-Kula (Uniwersytet Wrocławski)**
  Lévy-Khintchine type decomposition on universal compact quantum groups
  Lévy-Khintchine formula provides a classification of convolution semigroups of probability measures, or equivalently, of Lévy processes on $\mathbb{R}^n$ in terms of their genererator. It's generalization onto Lie groups is the Hunt formula. They both show how the generators of Lévy processes are combinations of continuous (or Gaussian) parts and jump parts.

  In 1990s M. Schürmann studied the notion of Lévy processes and related objects in the framework of bialgebras. He proved that an analoguous decomposition into maximal Gaussian and the remaining part holds on every commutative bialgebra, as well as on the Brown-Glockner-von Waldenfels algebra (the universal unital $*$-algebra generated by the coefficients of a unitary matrix). Later, Schürmann and Skeide showed that the decomposition is possible on $SU_q(2)$. Recently, this problem has received again some attention. In particular, Franz, Gelrhold and Thom provided the first example of a $*$-bialgebra which does not admit such a decomposition (the group algebra of the free product of $\mathbb{Z}^k$ with the wallpaper group "p2").

  We shall discuss the problem of the existence of the decomposition on the universal compact quantum groups $U_N^+(F)$ and $O_N^+(F)$, and show that there exists a family of quantum groups that does not admit the Lévy-Khintchine type decomposition and which are neither commutative nor cocommutative.

  This talk is based on the joint work with Biswarup Das, Uwe Franz and Adam Skalski.

- **Makoto Yamashita (Ochanomizu University)**
  Categorical duality for actions of compact quantum groups
  The Tannaka-Krein duality principle roughly says that a "group" can be reconstructed from its linear representations. A version due to Woronowicz captures this correspondence for the compact quantum groups formulated in the language of C^*-algebras. Building on this paradigm, this lecture series presents an analogous duality for "group actions", which correspond to the actions of tensor categories (Ostrik, Pinzari-Roberts, De Commer-Yamashita, Neshveyev). In recent years this viewpoint have led to various applications to concrete classification problems for compact quantum groups and their actions. The last part of the lecture will focus more on the recent development on analogy with the theory of subfactors and quantum symmetries. In particular, we will cover a dynamical characterization of the weak Morita equivalence, which generalizes work of Schauenburg and Bichon-De Rijdt-Vaes.

- **Sang-Gyun Youn (Seoul National University)**
  Similarity property of convolution algebras of locally compact quantum groups

  The celebrated work of Day and Dixmier in 1950 is equivalent to that every bounded representation of $L^1(G)$ is similar to a $*$-representation if G is amenable and the Dixmier's similarity problem is to prove the converse. In the category of locally compact quantum groups, M.Brannan, M.Daws and E.Samei showed that every completely bounded representation π of $L^1(\mathbb{G})$ with an additional condition $\|\check{\pi}\|_{cb} < \infty$ is similar to a $*$-representation if $\mathbb{G}$ is amenable. On the other hand, they showed that if $\mathbb{G}$ is a compact quantum group of Kac type, the assumption $\|\check{\pi}\|_{cb} < \infty$ can be removed.

  The purpose of our study is to show that the condition $\|\check{\pi}\|_{cb} < \infty$ is indispensible and to provide evidence that $\|\check{\pi}\|_{cb} < \infty$ can be removed if $\mathbb{G}$ is of Kac type. This talk is based on an ongoing joint work with M.Brannan.