On pathwise uniqueness of stochastic differential equations with Lévy noise

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Abstract

We consider the pathwise uniqueness of the d-dimensional stochastic differential equation

$$\begin{array}{l} dX_t = b(X_t)dt + \sigma(X_t)dW_t + \int_z F(X_{t^-}, z)d\bar{N}(dt, dz), \\ X_0 = x_0. \end{array} \tag{1}$$

Here $\bar{N} = N(dt, dz) - dtv(dz)$ and $N(dt, dz)$ is a Poisson random measure with compensator $dtv(dz)$. It is well known [2] that in the diffusion case (i.e., $F(\cdot) = 0$)), if $|b(x) - b(y)| \le c|x - y|r(|x - y|^2)$ and $|\sigma(x) - \sigma(y)| = \rho(|x-y|) \le c|x-y|\sqrt{r(|x - y|^2)}$ where $lim_{s \to 0} r(s) = \infty$ and $r(\cdot)$ satisfies the following :

$$\int_{0^+} ds/(sr(s)) = \infty \quad \text{and} \tag{G}$$

$$sr(s^2) \quad \text{is concave,} \tag{C}$$

then pathwise uniqueness holds for (1). The concavity condition (C) was later replaced in [1] by the growth condition

$$lim_{s \to 0}(r'(s)s)/r(s) = 0. \tag{G}$$

Indeed, condition (G) implies (C) if $lim_{s \to 0}(r'(s)s)/(r(s)) \uparrow 0$. However, in general they are not equivalent. In this talk, we include in (1) a Lévy noise $\bar{N}(dt, dz)$ and show that under the condition

$$\int_z |F(x, z) - F(y, z)|^2 v(dz) \le c|x - y|^2 r(|x - y|^2),$$

pathwise uniqueness holds. It is also well known that in the 1-dim diffusion case, condition (I) can be replaced by $\int_{0+} ds/\sqrt{\rho(s)} = \infty$. We shall show that in this case pathwise uniqueness holds under this weaker integrability condition and (G).

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References

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