ABSTRACTS

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Quasinormality of Hilbert Space Operators

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The class of bounded quasinormal operators was introduced by Brown[1] in 1953. Two different definitions of unbounded quasinormal operators appeared independently by Kaufman[5] in 1983 and (a few years later) by Stochel-Szafraniec[6] in 1989. In the present talk we show that both of these definitions coincide. We also discuss the question of whether the equality in Kaufman's definition of quasinormality can be replaced by inclusion. It is shown that the answer is in the affirmative if the inclusion is properly chosen. Next, we characterize quasinormality of unbounded operators in terms of the truncated operator Stieltjes moment problem. This part of our results is inspired by a result of Embry which characterizes quasinormality of bounded operators by means of the operator Stieltjes moment problem. It states that a closed densely defined operator C is quasinormal if and only if the equality $C^{*n}C^n = (C^*C)^n$ holds for $n = 2, 3$. In the case of bounded operators, in fact, this characterization has been known for specialists working in this area since late 1980's. Moreover, we discuss an absolute continuity approach to quasinormality of unbounded operators. On the way we characterize wider classes of operators that seem to be of independent interest. First we prove that a closed densely defined Hilbert space operator A is quasinormal if and only if $\langle E(\cdot) Af, Af\rangle \ll \langle E(\cdot) |A|f, |A|f\rangle$ for every vector f in the domain $\mathcal{D}(A)$ of A (the symbol means absolute continuity), where E is the spectral measure of the modulus $|A|$ of A. One may ask whether reversing the above absolute continuity implies the quasinormality of A. In general the answer is negative. Another question is: assuming more, namely that the Radon–Nikodym derivative of $\langle E(\cdot) |A|f, |A|f\rangle$ with respect to $\langle E(\cdot) Af, Af\rangle$ is bounded by a constant c which does not depend on $f \in \mathcal{D}(A)$, is it true that A is quasi-normal? We shall prove that the answer is affirmative for $c \leq 1$ and negative for $c > 1$. The case of $c > 1$ leads to a new class of operators, called weakly quasinormal, which are characterized by means of the strong commutant of their moduli. Let us remark that operators which satisfy the reversed absolute continuity condition can be completely characterized in the language of operator theory. The absolute continuity approach is implemented in the context of weighted shifts on directed trees (see [2] for definition and fundamental properties of weighted shifts on directed trees). This enables us to illustrate the theme of this article by various examples and to show that there is no relationship between the hyponormality class and the classes of operators studied in the present paper. Finally, we will provide some generalizations of our main results which cover the case of the so-called q-quasinormal operators. (This talk is based on [3] and [4].)

References

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The mean transform of bounded linear operators

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In this talk we introduce the mean transform of bounded linear operators acting on a complex Hilbert space and then explore how the mean transform of weighted shifts behave, in comparison with the Aluthge transform. (Joint work with W.Y. Lee and J. Yoon)

Hyperinvariant subspace problem for operators having a part

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We will introduce the notion of extremal vectors and apply it to the hyperinvariant subspace problem for operators having a part.

References

[1] S. Ansari and P. Enflo, *Extremal vectors and invariant subspaces*, Trans. Amer. Math. Soc. **350** (1998), 539-558.
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On adjacency operators associated by weighted directed graphs

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In 1989, M. Fujii, H. Sasaoka and Y. Watatani studied adjacency operators on infinite directed graphs and developed some relations between graphs and adjacency operators in the case that adjacency operators were bounded. In this talk, we introduce weighted adjacency operators on infinite directed graphs which generalizes adjacency operators on infinite directed graphs. We discuss some properties of weighted adjacency operators. In addition, some connections between weighted adjacency operators and graphs are studied and in particular we characterize the normality of weighted adjacency operators in the case that weighted adjacency operators are bounded. And we will discuss other operator properties such as hyponormality, quasinormality, etc. (Joint work with G. Exner, I.B. Jung and E. Y. Lee)

Concrete solution of the nonsingular quartic moment problem

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Given real numbers

$$\beta \equiv \beta^{(4)} : \beta_{00}, \beta_{10}, \beta_{01}, \beta_{20}, \beta_{11}, \beta_{02}, \beta_{30}, \beta_{21}, \beta_{12}, \beta_{03}, \beta_{40}, \beta_{31}, \beta_{22}, \beta_{13}, \beta_{04},$$

with $\beta_{00} > 0$, the *quartic real moment problem* for β entails finding conditions for the existence of a positive Borel measure μ, supported in $\mathbb{R}^2$, such that $\beta_{ij} = \int s^i t^j d\mu$ $(0 \le i + j \le 4)$. Let $\mathcal{M}(2)$ be the moment matrix for $\beta^{(4)}$, given by $\mathcal{M}_{\mathbf{i},\,\mathbf{j}} := \beta_{\mathbf{i}+\mathbf{j}}$, where $\mathbf{i}$, $\mathbf{j} \in \mathbb{Z}_+^2$ and $|\mathbf{i}|$, $|\mathbf{j}| \le 2$. Assume that the 6×6 moment matrix $\mathcal{M}(2)$ is nonsingular. In previous work, L. Fialkow and J. Nie proved, abstractly, the existence of a representing measure for $\mathcal{M}(2)$; since it is known that $\operatorname{card} \operatorname{supp} \mu \le \dim \mathcal{P}_{2n}$, where $\mathcal{P}_{2n} = \{f \in \mathbb{R}[x, y] : \deg f \le 2n\}$, this immediately implies that there exists one such representing measure with at most 15 atoms. In this talk we find concrete representing measure; moreover, we prove that it is possible to ensure that one such representing measure for $\mathcal{M}(2)$ is 6-atomic.

Embedding 2-variable weighted shifts

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In this talk, we explain several properties of embedding 2-variable weighted shifts. This is a joint work with Raul E. Curto and Sang Hoon Lee.

Rank of truncated Toeplitz operators with L^∞ symbols

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The compression of Toepltiz operators to the subspace $H^2 \ominus \theta H^2$ are called truncated Toeplitz operators. The rank formula for truncated Toeplitz operator with L^∞ symbols are given in terms of the shape of the symbols.

Subnormal and quasinormal Toeplitz operators

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In this talk we consider the subnormality and the quasinormality of Toeplitz operators with matrix-valued rational symbols. In particular, in view of Halmos's Problem 5, we focus on the question: Which subnormal Toeplitz operators are normal or analytic? Firstly, we prove that: Let $\Phi \in L^\infty_{M_n}$ be a matrix-valued rational function having a "matrix pole," i.e., there exists $\alpha \in \mathbb{D}$ for which $\ker H_\Phi \subseteq (z-\alpha)H^2_{\mathbb{C}^n}$, where H_Φ denotes the Hankel operator with symbol Φ. If

(i) T_Φ is hyponormal;
(ii) $\ker [T^*_\Phi, T_\Phi]$ is invariant for T_Φ,

then T_Φ is normal. Hence in particular, if T_Φ is subnormal then T_Φ is normal. Secondly, we show that every pure quasinormal Toeplitz operator with a matrix-valued rational symbol is unitarily equivalent to an analytic Toeplitz operator.