Weighted estimates for bilinear Bochner-Riesz means at the critical index

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December 10, 2021

Abstract: In this talk, I shall discuss weighted and unweighted boundedness of bilinear Bochner-Riesz means.

The bilinear Bochner-Riesz means is defined as

$$\mathcal{B}^{\alpha}(f,g)(x) := \int_{\mathbb{R}^n}\int_{\mathbb{R}^n}(1-|\xi|^2-|\eta|^2)_+^{\alpha}\hat{f}(\xi)\hat{g}(\eta)e^{-2\pi\iota x\cdot(\xi+\eta)}d\xi d\eta,$$

where α is a complex number with non-negative real part and f, g are Schwartz class functions and $r_+ = r$, if $r > 0$ and 0 elsewhere.

One can observe that the critical index of this operator is $n-\frac{1}{2}$. i.e. the operator $\mathcal{B}^{\alpha}$ maps $L^{p_1}(\mathbb{R}^n)\times L^{p_2}(\mathbb{R}^n)$ to $L^{p}(\mathbb{R}^n)$, for all $1 \leq p_1, p_2 \leq \infty$ with $\frac{1}{p_1}+\frac{1}{p_2}=\frac{1}{p}$, if $Re(\alpha) > n-\frac{1}{2}$. It was known that for $Re(\alpha) > n-\frac{1}{2}$, the operator $\mathcal{B}^{\alpha}$ is pointwise dominated by the product of Hardy-Littlewood maximal operators and from there one can get the weighted boundedness for the product type Muckenhoupt weight classes $A_{p_1}\times A_{p_2}$. In this talk, I shall prove weighted boundedness of the operator $\mathcal{B}^{n-\frac{1}{2}}$, for bilinear weight class $A_{\vec{P}}$, which is a bigger class than $A_{p_1}\times A_{p_2}$ and $\vec{P}=(p_1,p_2)$, $1 < p_1, p_2 \leq \infty$.