Vector 1, historical background

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Vector, Scalar

Scalar $\leftarrow$ scale   magnitude

  field: $+, -, \times, \div$

e.g. $\mathbb{R}, \mathbb{C}$

Vector $\leftarrow$ vectus $\leftarrow$ vehere   magnitude and direction

  $+, -$, scalar multiplication

$\mathbb{C}$, $\mathbb{H}$

basis of vector spaces

basis
dimension
e.g. $\mathbb{R}^n$

additional operations:

dot product=inner product, $\mathbf{A} \cdot \mathbf{B}$

cross product $\mathbf{A} \times \mathbf{B}$

Newtonian mechanics

17C

Isaac Newton (1643-1727)

Work $W = \mathbf{F} \cdot \mathbf{S}$
Torque $\mathbf{T} = \mathbf{L} \times \mathbf{F}$

vector fields

19C, 20C
Electro magnetism
Gradient, Divergence, Curl

James Clerk Maxwell (1831-1879)

Maxwell's equations

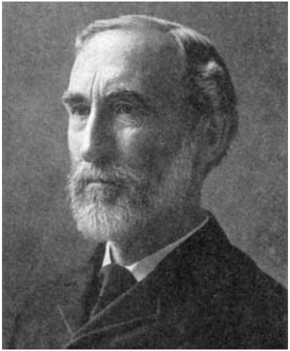
$\vec{E}(x,y,z,t)$, $\vec{H}$, current density $\vec{J}$, charge density ρ

$$\vec{\nabla}\cdot\vec{E}=\rho \quad \text{(Gauss' law)}$$
$$\vec{\nabla}\cdot\vec{H}=0 \quad \text{(no magnetic sources)}$$
$$\vec{\nabla}\times\vec{E}+\frac{\partial\vec{H}}{\partial t}=0 \quad \text{Faraday's law)}$$
$$\vec{\nabla}\times\vec{H}-\frac{\partial\vec{E}}{\partial t}=\vec{J} \quad \text{(Ampère's law).}$$

Hamilton

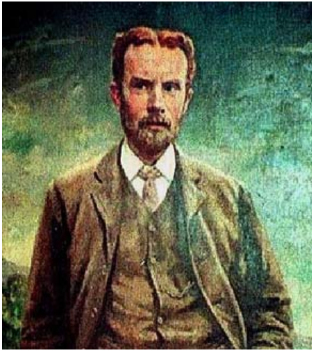
William Rowan Hamilton (1805-1865)

Gibbs



Josiah Willard Gibbs (1839-1903)

Heaviside



Oliver Heaviside (1850-1925)

a self-taught English electrical engineer, mathematician, and physicist who adapted complex numbers to the study of electrical circuits, invented mathematical techniques to the solution of differential equations (later found to be equivalent to Laplace transforms), reformulated Maxwell's field equations in terms of electric and magnetic forces and energy flux, and independently co-formulated vector analysis. Although at odds with the scientific establishment for most of his life, Heaviside changed the face of mathematics and science for years to come.

divergence theorem

In $\mathbb{R}^3 = \{(x_1, x_2, x_3)\}$ let $\vec{F} = (f_1, f_2, f_3)$ be a vector field

$\text{div}\vec{F} = \vec{\nabla} \cdot \vec{F} := \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3}$

Divergence theorem:

$$\iiint_{\Omega} \vec{\nabla} \cdot \vec{F} \, dV = \iint_{\partial\Omega} \vec{F} \cdot \vec{n} \, d\sigma.$$

curl, Stokes theorem

In $\mathbb{R}^3 = \{(x_1, x_2, x_3)\}$ let $\vec{F} = (f_1, f_2, f_3)$ be a vector field

$$\text{curl } \vec{F} = \vec{\nabla} \times \vec{F} := \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

Stokes' theorem:

$$\iint_S \vec{\nabla} \times \vec{F} \cdot \vec{n} \; d\sigma = \int_{\partial S} \vec{F} \cdot \vec{T} \, ds.$$