Complex Analysis Midterm Exam

October 27, 2001

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1. (10) Find all the third roots of i.

2. (20) Evaluate the integrals:

   a) $\int_{|z|=1} \frac{e^{3z}dz}{z+5}$.

   b)$\int_{|z|=1} \frac{e^{3z}dz}{z(z+5)}$.

   c) $\int_{|z|=1} \frac{e^{3z}dz}{z^2(z+5)}$.

3. (20) Let $f(z)$ be an analytic function whose real part is $u(x,y) = e^{-y}\cos x$.

   a) Find the imaginary part of $f(z)$.

   b) Find $f'(z)$.

4. (60) Prove :

   a)If $u(x,y)$ is harmonic then $u_x - iu_y$ is holomorphic.

   b) If $f(z) = u + iv$ is analytic in a domain and $f'(z) \neq 0$ then the level curves of u and v are orthogonal to each other.

   c) If $u(x,y)$ is harmonic and $u(x,y) > 0$ for all $(x,y) \in \mathbf{C}$ then u is constant.

   d) If an entire function f satisfies $|f(z)| < A|z|^n$ for some positive number A then f is a polynomial of degree $\leq n$.

   e) $|\int_{|z|=1} \frac{dz}{3+5z^2}| \leq \pi$.

   f) If $f(z)$ is analytic in a domain such that $|f(z)| =$ constant then $f(z)$ is constant.

5. (15) Evaluate $\int_C \frac{dz}{z}$, where C is a contour given by

$$z(t) = 3t - t^2 + (1-t^2)i, \quad 0 \leq t \leq 1.$$

6. (15) Find the point(s) where $|\cos z|$ attains its maximum in the closed rectangle $0 \leq x \leq 2\pi$, $0 \leq y \leq 2\pi$. What is the maximum value?