미분기하 중간고사 (2003년 4월 23일)
*표 문제는 제출하지 마시오.

1(20pts). Consider a curve $\alpha(t) = (t, \quad \cosh t, \quad 1+t)$ in $\mathbb{R}^3$ and real valued functions $f(x,y,z) = \ln(x^2+y^2+z^2), \quad g(x,y,z) = x - 4y + e^z$.
a) Evaluate $\frac{d}{dt}(f \circ \alpha) \quad$ at $t = 0$.
b) Let $F = (f, g) : \mathbb{R}^3 \to \mathbb{R}^2$ and let $\beta(t) = F(\alpha(t))$. Find $\beta'(0)$.

*2. Let $\alpha(t) = (a\cos t, \quad a\sin t, \quad bt)$.
a) Reparametrize α by arclenth s.
b) Find the Frenet frame, the curvature and the torsion of the curve.

3(10pts). Suppose that α is a curve in $\mathbb{R}^3$ with constant speed v. Prove that the acceleration of the curve is kv^2N, where k is the curvature and N is the normal vector field to the curve.

4(20pts). On $\mathbb{R}^2$ consider the frame field
$E_1 = \cos\phi U_1 + \sin\phi U_2, \quad E_2 = -\sin\phi U_1 + \cos\phi U_2$.

a) Show that $\theta_1 = dr, \quad \theta_2 = rd\phi$ are dual 1-forms, where $\phi = \tan^{-1}\frac{y}{x}$ and $r = \sqrt{x^2+y^2}$.

b) For $i, j = 1, 2$ find the 1-forms ω_{ij} that satisfies $\omega_{ij}(v) = \nabla_v E_i \circ E_j(P)$ for any tangent vector v to $\mathbb{R}^2$ at the point P.

*c)What are the structural equations? Answer the same questions for the spherical frame.

5(20pts). Suppose that $\{E_1, E_2, E_3\}$ is a frame field on $\mathbb{R}^3$ and A is its attitude matrix. Let $\omega_{ij}, \quad i, j = 1, 2, 3$ be the connection form associated with the frame field, which is defined by

$$\omega_{ij}(v) = \nabla_v E_i \circ E_j(P)$$

for any tangent vector v to $\mathbb{R}^3$ at P.

a) Show that $\omega_{ji} = -\omega_{ij}$.

b) Show that the 3×3 matrix (ω_{ij}) is given by $(dA)({}^tA)$.

6(20pts). If $F : \mathbb{R}^3 \to \mathbb{R}^3$ is a diffeomorphism such that F_* preserves dot product, show that F is an isometry.

7(10pts). Let a be a point of $\mathbb{R}^3$ such that $|a| = 1$. Prove that $C(p) = a \times p + (p \circ a)a$ defines an orthogonal transformation.

*8. If two unit-speed curves have the same curvature and the same torsion then they are congruent.

9(30pts). Let $\beta(t) = \frac{1}{2\sqrt{2}}(t + \sqrt{3}\sin t, \quad 2\cos t, \quad \sqrt{3}t - \sin t)$ and $\alpha(t) = \frac{1}{\sqrt{2}}(\cos t, \sin t, -t)$.

a) Show that α and β are congruent.

b) Find an isometry F such that $F(\beta) = \alpha$.

총 130점 끝