복소함수론 학기말시험

2006. 12. 14

1 (15 pts) Evaluate the integral: $\int_{|z|=2} f(z)dz$, where $f(z) = \frac{1}{z(z-1)(z-3)}$.

2 (15 pts) Suppose that an entire function $f(z)$ satisfies

$$|f(z)| \leq A + B|z|^k, \quad \forall z \in \mathbb{C},$$

for some positive numbers A and B and for some positive integer k. Show that $f(z)$ is a polynomial of degree $\leq k$.

3 (15 pts) Suppose that f is analytic in the annulus $A = \{z : 1 \leq |z| \leq 2\}$ and $|f(z)| \leq 1$ for $|z| = 1$, $|f(z)| \leq 4$ for $|z| = 2$. Prove that $|f(z)| \leq |z|^2$ for all $z \in A$.

4 (30 pts = 15+15) Let $f(z) = z^5 - \frac{1}{2}z^4 + 2z^3 + 1$.

a) Show that $f(z)$ has five zeros in $|z| < 2$.

b) Evaluate $\int_{|z|=2} \frac{zf'(z)}{f(z)}dz$.

5 (15 pts) Let C denote the positively oriented circle $|z| = 8$. Show that

$$\frac{1}{2\pi i}\int_C \frac{e^{zt}}{\sinh z}dz = 1 - 2\cos \pi t + 2\cos 2\pi t.$$

6 (30 pts = 15+15)

a) Find the inverse Laplace transform of $\frac{1}{s(s-2)^2}$. Use the residue theorem and show the convergence of the integral.

b) Using the method of Laplace transform find the solution $y(t)$ of the initial value problem:

$$y'' - 4y' + 4y = \begin{cases} 1, & \text{if } t > 1 \\ 0, & \text{if } t \leq 1, \end{cases}$$
$$y(0) = 1, \quad y'(0) = -2.$$

뒷면에 계속

7 (30 pts = 15+15) Let $f(z) = u(x,y) + iv(x,y)$ be an entire function such that $f'(z) \neq 0, \quad \forall z \in \mathbb{C}$. Show that

a) The level curves of u and the level curves of v intersect perpendicularly.

b) If $h(x,y)$ is a harmonic function such that $h(x,y) \geq u(x,y)$ for all $(x,y) \in \mathbb{R}^2$, and $h(0,0) = u(0,0)$, then $h = u$.

8 (15 pts) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^z}$ converges absolutely for Re $z > 1$.

9 (15 pts) Prove: Suppose that for each $n = 1, 2, \cdots$, $f_n(z)$ is analytic in a simply connected domain D, and that the sequence $\{f_n(z)\}$ converges to $f(z)$ uniformly in D. Then $f(z)$ is analytic.

문제 끝

총점 180 점