복소함수론 중간 고사
2006년 10월 26일

1. (20 점) True or False? Give a brief explanation.

a) $e^{\log z} = z, \quad$ for $z \neq 0$.
b)$\log e^z = z$.
c)$\mathrm{Log}(1+i)^2 = 2\mathrm{Log}(1+i)$.
d) $\mathrm{Log}(\sqrt{2}+i)^4 = 4\mathrm{Log}(\sqrt{2}+i)$.

2. (10 점) Find a point z for which $\sin z = 2$.

3.(10 점)Evaluate i^i.

4. (15 점) Suppose that $f(z) = u(z) + iv(z)$ is an an entire function and that u is a function of x alone. Show that $f(z) = az + b$, where a and b are constants and $a \in \mathbb{R}$.

5.(20 점) a) (Cauchy's theorem) Suppose that f is analytic on and within a simple closed contour C. Assuming $f \in C^1$ prove that

$$\int_C f(z)dz = 0.$$

b)State and prove Morera's theorem.

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6.(40점) Let C be a positively oriented rectangle with vertices $-2+i, -2-i, 3-i, 3+i$. Evaluate the following integrals:

a) $\int_C z^n dz$, where $n = 0, \pm 1, \pm 2, \cdots$.

b) $\int_C (z^2 + \bar{z}) dz$.

c) $\int_C \frac{e^{3z}}{z^2(z+5)} dz$.

d) $\int_C \frac{e^{3z}}{z(z+1)} dz$.

7. (15 점)Suppose that $u(x, y)$ is the real part of an entire function $f(z)$ and that u is bounded from below, that is, $u(x, y) > M$, for some constant M. Show that u is constant.

8.(15+5=20 점) a) For complex numbers a and b with $|a| \leq 1$ and $|b| \leq 1$ show that

$$\left| \frac{a-b}{1-\bar{a}b} \right| \leq 1.$$

b) When does the equality hold in a)?

광고: 11월30일 목요일 19시-22시, 실함수의 적분에 관한 실기 테스트

150점 만점. 끝