Complex analysis (Grad) Quiz 1

October 7th, 2008

1 If f' exists at z_0 then f is continuous at z_0.

2 Suppose that $f(z)$ is holomorphic on a closed curve γ. Show that

$$\int_\gamma \overline{f(z)} f'(z) dz$$

is purely imaginary.

3 Compute

$$\int_{|z|=\rho} \frac{|dz|}{|z-a|^2}$$

under the condition $|a| \neq \rho$.

4 If $f(z)$ is analytic in the closed rectangle $R = [a, b] \times [c, d]$. Without assuming the continuity of $f'(z)$ prove that $\int_{\partial R} f(z)dz = 0$.

5 State and prove the Schwarz lemma.

6 Determine explicitly the largest disk about the origin whose image under the mapping $w = e^z$ is one-to-one.

7 If $f(z)$ is a holomorphic mapping of closed unit disk $|z| \leq 1$ into itself. Show that

$$\frac{|f'(z)|}{1-|f(z)|^2} \leq \frac{1}{1-|z|^2}.$$

8 Find the poles and residues of $\cot z$.

9 Evaluate

$$\int_0^\infty \frac{x^{1/3}}{1+x^2} dx.$$

10 (Bergman kernel) If $f(z)$ is holomorphic and bounded for $|z| < 1$ and if $|\zeta| < 1$, then

$$f(\zeta) = \frac{1}{\pi} \iint_{|z|<1} \frac{f(z) dx dy}{(1-\bar{z}\zeta)^2}.$$

End of problem set .