금 융 수 학 1 - Quiz 1

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1. In a single-period model, assume there are two states ω_1 and ω_2 at time $t = 1$. Assume the interest rate is 0.1. Let S_t $(t = 0, 1)$ be the stock price process given by

$$\begin{array}{ccc} & & S_1(\omega_2) = 132 \\ S_0 = 100 & \begin{array}{c} \nearrow \\ \searrow \end{array} & \\ & & S_1(\omega_1) = 66 \end{array}$$

Let X be a contingent claim such that $X(\omega_1) = -23.1$ and $X(\omega_2) = 16.5$. Answer the following questions. (each 10 points)

(a) (10pts) Find the replicating portfolio.

(b) (10pts) Find the martingale measure Q.

(c) (10pts) Compute the value at $t = 0$ of the option X.

(d) (10pts) Suppose the market price at $t = 0$ of the option X is 10. You can engage in an arbitrage to generated a risk-free profit. What is the profit and explain how you can achieve it.

2. Suppose there are four states $\omega_1, \omega_2, \omega_3$ and ω_4 in a single-period model. Assume the interest rate during this period is 0.2. Let the stock price S_t $(t = 0, 1)$ process is given by $S_0 = 48$ and $S_1(\omega_1) = 12, S_1(\omega_2) = 24, S_1(\omega_3) = 36, S_1(\omega_4) = 60$. Let X be an option given by $X(\omega_1) = 36, X(\omega_2) = 12, X(\omega_3) = 96$ and $X(\omega_4) = 12$. Answer the following question.

(a) (10pts) Is X attainable? Give reasons for your answer.

(b) (10pts) Find a martingale measure.

(c) (10pts) Is this martingale measure unique?

(d) (10pts) Draw graphically the buyer's and the seller's prices.

(e) (10pts) Suppose Y is another contingent claim $Y(\omega_1) = 36, Y(\omega_2) = 24, Y(\omega_3) = 12$ and $Y(\omega_4) = -12$. Is Y attainable? If so, explain graphically.

(f) (10pts) Compute $E_Q[Y^*] = E_Q[Y/B_1]$

(g) (10pts) Is the value computed above the value at $t = 0$ of Y? If so, why?

(h) (10pts) Let R be another martingale measure. Compute $E_R[Y^*]$.