# Entrance Exam. (Geometry & Topology for Ph.D. Course)

2014. 5. 2.

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1. Find all Betti numbers of the 3-dimensional torus $T^3$.

2. What is the dimension of the Grassmann manifold of planes in 5-dimensional Euclidean space?

3. Where is the center of mass of a tetrahedron?

4. Answer 'Y' if yes, or 'N' if no.

   (i) (    ) Is the unit tangent bundle of $S^2$ diffeomorphic to $SO_3$?

   (ii) (    ) Is $SO_3$ diffeomorphic to the projective space $P^3$ ?

   (iii) (    ) Is the projective space $P^4$ orientable?

   (iv) (    ) Is the antipodal map on the 2-sphere orientation preserving?

   (v) (    ) Is the antipodal map on the 3-sphere orientation preserving?

   (vi) (    ) If $f : \mathbb{R}^2 \to (\mathbb{R}^2 - \{(0,0)\})$ is a continuous map, do there exist continuous maps $a, b : \mathbb{R}^2 \to \mathbb{R}$ such that

   $$f(x,y) = (a(x,y)\cos(b(x,y)), a(x,y)\sin(b(x,y)))$$

   for all $(x,y) \in \mathbb{R}^2$.

   (vii) (    ) Can you integrate a real valued continuous function defined on a compact manifold?

   (viii) (    ) Is any integral curve of any vector field on a compact smooth manifold without boundary defined for all real numbers?