기하학 고사

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1. Let $\gamma(t)$ be a regular curve in $\mathbb{R}^3$ with the curvature k at $t = 0$. Let $S : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map given by the matrix

$$3 \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

What is the curvature of the curve $S(\gamma(2t)) + (0,0,1)$ at $t = 0$?

2. Let κ denote the curvature function of the plane curve Γ given by the equation $x^2 + 4y^2 = 1$. Compute the total curvature $\int_\Gamma \kappa\, ds$.

3. Let S be the surface in $\mathbb{R}^3$ given by the equation

$$x^2 + y^2 - z^2 = 1.$$

   (a) Draw the surface S.

   (b) Does S have a point with zero Gaussian curvature? Does S have a point with positive Gaussian curvature?

4. Let $H = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1,\ z > 0\}$ be the hemisphere.

   (a) Does there exist a conformal (i.e., angle preserving) map from H onto an open subset of $\mathbb{R}^2$?

   (b) Does there exist an equi-areal (i.e., area preserving) map from H onto an open subset of $\mathbb{R}^2$?

   (c) Does there exist an isometry from H onto an open subset of $\mathbb{R}^2$?