집합과 수리논리: 중간 고사 2013 년 4 월 22 일 (월) 11:00–12:15 (총점 104 점)

소속: 학번: 이름: 점수:

배점: 문 1∼ 6 (Need Justifications): 각 10점,
문 7: 8점, 문 8: 각 2점(오답: 0점, 무답 1점, 정답 2점)

1. Show that for any set A, there exists no bijection of A onto $\mathcal{P}A$.

2. Let A be a set of disjoint open disks in the plane $\mathbb{R}^2$. Is A countable?

3. Does $x^+ = y^+$ imply $x = y$?

4. When $\bigcup x = \bigcap x$?

5. Let $(A, <)$ be a well-ordered set. For any $x \in A$, let $A_x := \{y \in A \mid y < x\}$. Suppose S is a nonempty subset of A such that for any $x \in A$, $A_x \subset S$ implies $x \in S$. Is $S = A$?

6. Show that $\mathcal{P}(\mathbb{N}) \sim \mathbb{R}$.

7. Let w be the ordered pair (a, b). Simplify the following.

(a) $\bigcup w$ _____ (e) $\bigcup\bigcap w$ _____

(b) $\bigcup\bigcup w$ _____ (f) $\bigcap\bigcup w$ _____

(c) $\bigcap w$ _____ (g) $\bigcup w - \bigcap w$ ________

(d) $\bigcap\bigcap w$ _____ (h) $\bigcap w - \bigcup w$ _____

8. In '( )' write T if true, F if false.

( ) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

( ) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

( ) $A - (B - C) = (A - B) \cup (A \cap C)$

( ) $(A - B) - C = A - (B \cup C)$

( ) $(A \cup B) - C = (A - C) \cup (B - C)$

( ) $\bigcup \varnothing = \varnothing$

( ) For any set A, $A^{\varnothing} = \{\varnothing\}$.

( ) For any set A, there exists no injection from $\mathcal{P}A$ into A.

( ) For any set A, $A \subseteq \mathcal{P}(\bigcup A)$.

( ) For any set x, there exists a set y such that $x \in y$.

( ) For any set x, there exists a set y such that $y \notin x$.

( ) There exists a set A of open intervals in $\mathbb{R}$ such that $\mathbb{Q} \subseteq \bigcup A \subset \mathbb{R}$.

( ) There is a bijection from the Cantor's dust onto $\mathbb{R}^\infty = \mathbb{R} \cup \mathbb{R}^2 \cup \mathbb{R}^3 \cup \cdots$.

( ) If f be a function defined on a natural number $n = \{0, 1, \ldots, n-1\}$, then $\prod f \sim f(0) \times f(1) \times \cdots \times f(n-1)$.

( ) The axiom of choice is equivalent to the following assertion: If E is an equivalence relation on a set A and if $\pi : A \to A/E$ is the canonical projection, then there exists a function $s : A/E \to A$ such that $\pi \circ s = \mathrm{id}_{A/E}$.

( ) $\mathcal{P}A = \mathcal{P}B$ implies $A = B$.

( ) $\bigcup A = \bigcup B$ implies $A = B$.

( ) $a \in B$ implies $\mathcal{P}a \in \mathcal{P}B$.

( ) I love Set Theory.

모범답안

[2]Yes, since each disk contains a rational point.

[3]Supposer $x^+ = y^+$. If $x \neq y$, then we have $x \in y$ and $y \in x$. Thus the set $\{x, y\}$ is not well-founded. Thus the axiom of foundation implies $x = y$.

[4]Note that x cannot be the empty set. It holds if and only if x is a singleton. For, if x has distinct elements a and b, then there exists an element $y \in (a - b) \cup (b - a)$. Thus we have $y \in \bigcup x - \bigcap x$, a contradiction.

[5]NO, because .... If '$\subset$' is replaced by '$\subseteq$', then YES, because ....

[7](a) $\{a, b\}$ (b) $a \cup b$ (c) $\{a\}$ (d) a (e) a (f) $a \cap b$ (g) $\{b\}$ if $a \neq b$, $\varnothing$ if $a = b$. (h) $\varnothing$

[8]T, T, ..., T, F, F, ?