HW 2

Due March 20, 2013

Some Notations

- For sets A and B, let

  $$A - B := \{x \mid x \in A \ \&\ x \notin B\}.$$

  We write

  $$A \subseteq B \quad \text{or} \quad B \supseteq A$$

  if every element of A is an element of B. In this case we write

  $$A \subset B \quad \text{or} \quad B \supset A$$

  if $A \neq B$.

- If f is a function, then

  $$f[A] := \{f(a) \mid a \in A\}.$$

- If f is a function, then

  $$f^1 = f, \quad f^2 := f \circ f, \quad f^3 := f \circ f \circ f, \quad \ldots$$

  and f^0 is the identity map.

- Let B^A be the set of all functions from A into B.

Homeworks 2

1. Show that $f[A - B] \supseteq f[A] - f[B]$.

2. Show that, if f is injective, $f[A - B] = f[A] - f[B]$.

3. Let h be a function from A into itself. Let

$$A_n := h^n[A] - h^{n+1}[A]$$

for $n = 0, 1, 2, \ldots$, and let

$$h^\infty[A] = \bigcap_{n=0}^{\infty} h^n[A].$$

Show that

$$A - \bigcup_{n=0}^{\infty} A_n = h^\infty[A].$$

4. For sets A, B and C, show that

$$(A^B)^C \sim A^{B \times C}.$$

5. Show that the set of all infinite sequences of $\mathbb{R}$ is equipotent to $\mathbb{R}$, i.e., $\mathbb{R}^{\mathbb{N}} \sim \mathbb{R}$.