Qudit

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JK, J. Lee, S.-W. Ji, H. Nha, P. Anisimov, J. P. Dowling,
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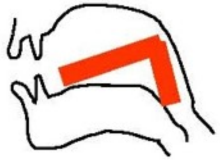
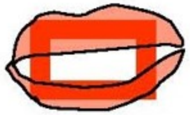
KOREAN Characters "Hangul": Consonants and Vowels



ㄱ = g or k — beginning of a word; after non-vocal sound; last sound of a syllable
ㄲ = k'
ㅋ = k

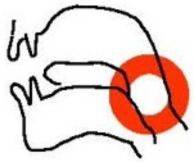
ㄴ = n
ㄷ = d or t — beginning of a word; after non-vocal sound; last sound of a syllable
ㄸ = d'
ㅌ = t
ㄹ = r or l — last sound of a syllable



ㅁ = m
ㅂ = b or p — beginning of a word; after non-vocal sound; last sound of a syllable
ㅃ = p'
ㅍ = p

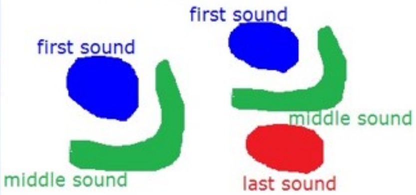
ㅅ = s
ㅆ = s'
ㅈ = j
ㅉ = j', ts
ㅊ = ch, ts

(ㅅ, ㅆ, ㅈ, ㅉ, ㅊ) t — last sound of a syllable

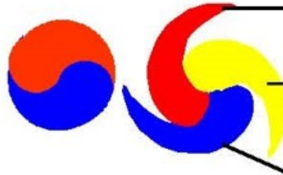


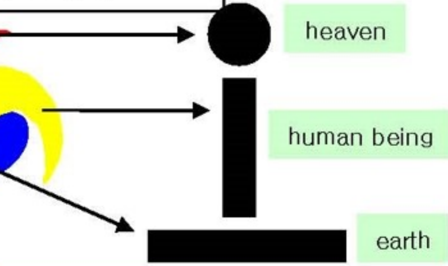
ㅇ = no sound — first sound of a syllable
ㅇ = ng — last sound of a syllable
ㅎ = h or t — last sound of a syllable

KOREAN syllables



세 종 대 왕
Se jong Dae wang
King Sejong the Great, inventor of Hangul





Mnemonic for vowels
ㅏ = dot **A**fter the bar (German A)
ㅓ = dot **O**cc**U**rs before the bar
ㅗ = dot **O**ver the bar (German O)
ㅜ = dot **U**nder the bar (German U)
ㅣ = like **I**
ㅑ = ya; 2 dots for **Y** sound

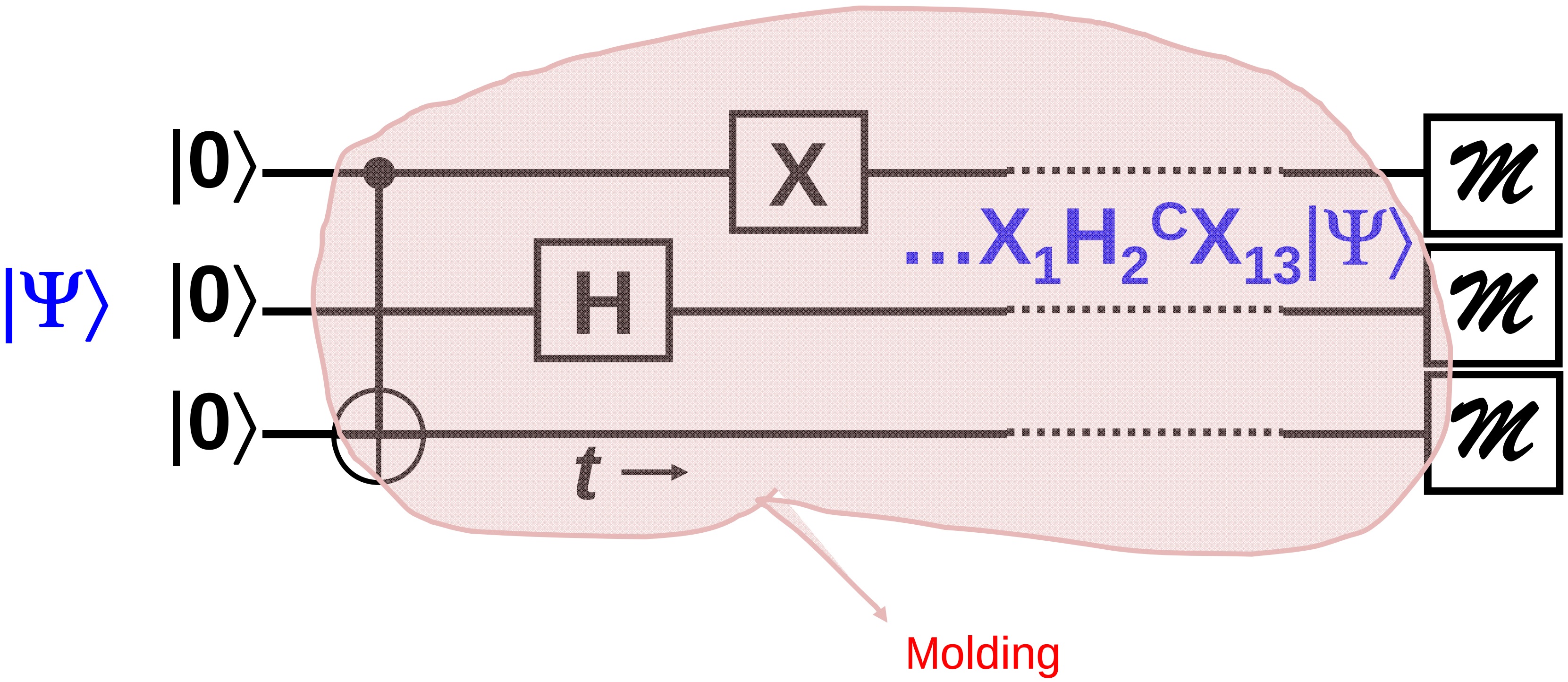
ㅏ a	ㅐ ae		
ㅑ ya	ㅒ yae		
ㅓ eo	ㅔ e		
ㅕ yeo	ㅖ ye		
ㅗ o	ㅘ wa	ㅙ wae	ㅚ oe
ㅛ yo			
ㅜ u,oo	ㅝ wo	ㅞ we	ㅟ wee, ue
ㅠ yu,yoo			
ㅡ eu, u(short)	ㅢ eui, ui		
ㅣ i, ee			

Thank you. = 감사합니다 (kam sa hap ni da > kam sa ham ni da)
고맙습니다 (ko map sup ni da > ko map sum ni da)
greetings = 안녕하세요 (an nyeong ha se yo)
good bye = 안녕히 계세요 (an nyeong hi gye se yo) (Stay with peace.)
안녕히 가세요 (an nyeong hi ga se yo) (Go with peace.)

Write your name here.

Your name in Hangul.

MOLDING a Quantum State



E. Knill, R. Laflamme, and G. J. Milburn, Nature 409, 46 (2001).

M. A. Nielsen, Phys. Rev. Lett. 93, 040503 (2004).

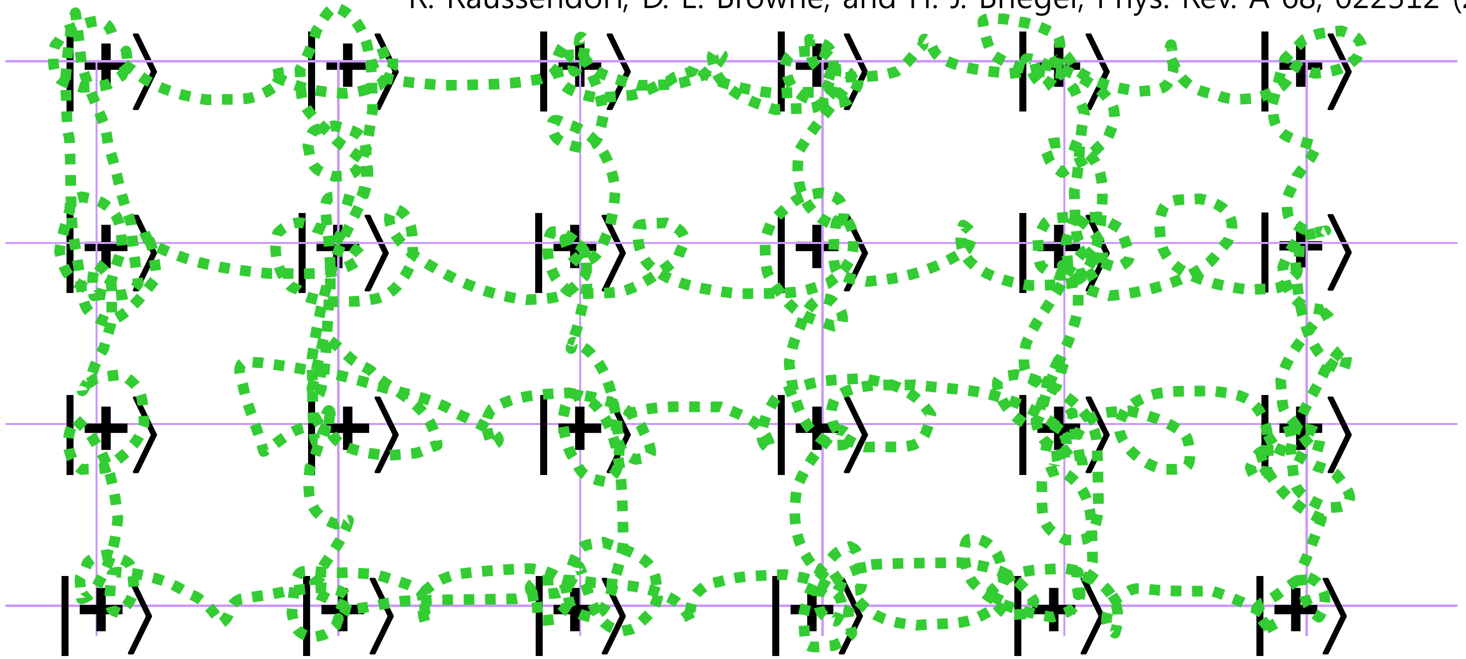
M. A. Nielsen and C. M. Dawson, Phys. Rev. A 71, 042323 (2005).

SCULPTURING a Quantum State

- Cluster State [One-way] Quantum Computing -

R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. 86, 5188 (2001).

R. Raussendorf, D. E. Browne, and H. J. Briegel, Phys. Rev. A 68, 022312 (2003).



1. Initialize each qubit in $|+\rangle=(|0\rangle+|1\rangle)/\sqrt{2}$ state.
2. Contolled-Phase(CZ) between the neighboring qubits.
3. Single qubit manipulations and single qubit measurements only [Sculpturing]. No two qubit operations!

Cont-Z and $|+\rangle$

- Commuting with each other
- Symmetric w.r.t. control and target

Even superposition
of computational basis states

$$Z_{12}|+\rangle_1|+\rangle_2 = Z_{12}\left(\frac{|0\rangle_1+|1\rangle_1}{\sqrt{2}}\right)\left(\frac{|0\rangle_2+|1\rangle_2}{\sqrt{2}}\right)$$

$$= Z_{12}\frac{|0\rangle_1}{\sqrt{2}}\left(\frac{|0\rangle_2+|1\rangle_2}{\sqrt{2}}\right) + Z_{12}\frac{|1\rangle_1}{\sqrt{2}}\left(\frac{|0\rangle_2+|1\rangle_2}{\sqrt{2}}\right)$$

$$= \frac{|0\rangle_1}{\sqrt{2}}\left(\frac{|0\rangle_2+|1\rangle_2}{\sqrt{2}}\right) + \frac{|1\rangle_1}{\sqrt{2}}\left(\frac{|0\rangle_2-|1\rangle_2}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}}|0\rangle_1|+\rangle_2 + \frac{1}{\sqrt{2}}|1\rangle_1|-\rangle_2 \quad \text{Bell State}$$

$$Z_{12}Z_{23}|+\rangle_1|+\rangle_2|+\rangle_3 = Z_{21}Z_{23}|+\rangle_1\frac{|0\rangle_2}{\sqrt{2}}|+\rangle_3 + Z_{21}Z_{23}|+\rangle_1\frac{|1\rangle_2}{\sqrt{2}}|+\rangle_3$$

$$= \frac{1}{\sqrt{2}}|+\rangle_1|0\rangle_2|+\rangle_3 + \frac{1}{\sqrt{2}}|-\rangle_1|1\rangle_2|-\rangle_3 \quad \text{GHZ state}$$

B. C. Sanders and G. J. Milburn, Phys. Rev. A 45, 1919 (1992).
M. Paternostra et al., Phys. Rev. A 67, 023811 (2003).
Wang W.-F. et al., Chin. Phys. Lett. 25, 839 (2008)
Nguyen B. A. and J. Kim, Phys. Rev. A 80, 042316 (2009).

An Interesting Observation of Exponential Function

$$e^x = \underbrace{1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^{d-1}}{(d-1)!}}_{d \text{ terms}}$$

$$+ \frac{x^d}{d!} + \frac{x^{d+1}}{(d+1)!} + \frac{x^{d+2}}{(d+2)!} + \cdots + \frac{x^{2d-1}}{(2d-1)!}$$

$$+ \frac{x^{2d}}{(2d)!} + \frac{x^{2d+1}}{(2d+1)!} + \frac{x^{2d+2}}{(2d+1)!} + \cdots + \frac{x^{3d-1}}{(3d-1)!}$$

$$+ \cdots$$

$$= f_0(x) + f_1(x) + f_2(x) + \cdots + f_{d-1}(x)$$

$$= \sum_{k=0}^{d-1} f_k(x)$$

$$f_k(x) = \sum_{m=0}^{\infty} \frac{x^{k+md}}{(k+md)!} \quad \text{for } k = 0, 1, \cdots, d-1$$

$$f_0(x) = 1 + \frac{x^d}{d!} + \frac{x^{2d}}{(2d)!} + \cdots$$

$$f_1(x) = \frac{x}{1!} + \frac{x^{d+1}}{(d+1)!} + \frac{x^{2d+1}}{(2d+1)!} + \cdots$$

$$\vdots$$

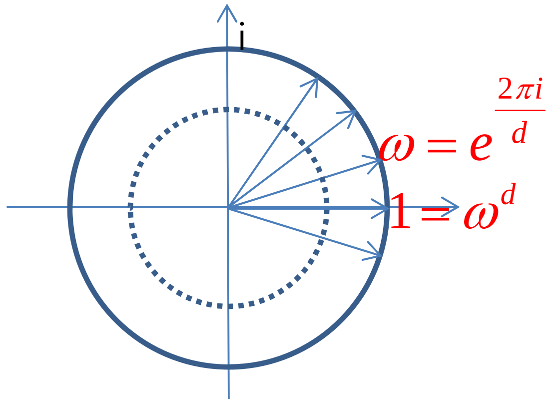
$$f_{d-1}(x) = \frac{x^{d-1}}{(d-1)!} + \frac{x^{2d-1}}{(2d-1)!} + \frac{x^{3d-1}}{(3d-1)!} + \cdots$$

$$f'_{d-1}(x) = f_{d-2}(x),\ f'_{d-2}(x) = f_{d-3}(x),\ \cdots,\ f'_1(x) = f_0(x),\ f'_0(x) = f_{d-1}(x)$$

As $x \to \infty$, $\dfrac{f_l(x)}{e^x} \to \dfrac{1}{d}$.

$$O(e^{-\frac{2\pi^2}{d^2}x})$$

$$\omega = e^{\frac{2\pi i}{d}} \qquad 1 = \omega^d$$



$$\sum_{s=0}^{d-1} \omega^{-ks} e^{\omega^s x} = \sum_{s=0}^{d-1} \omega^{-ks} \sum_{l=0}^{d-1} \sum_{m=0}^{\infty} \frac{(\omega^s x)^{l+md}}{(l+md)!}$$

$$= \sum_{s=0}^{d-1} \sum_{l=0}^{d-1} \sum_{m=0}^{\infty} \omega^{-ks} \omega^{sl} \underbrace{\omega^{smd}}_{=1} \frac{x^{l+md}}{(l+md)!}$$

$$= \sum_{l=0}^{d-1} \sum_{m=0}^{\infty} \frac{x^{l+md}}{(l+md)!} \underbrace{\sum_{s=0}^{d-1} \omega^{s(l-k)}}_{d \cdot \delta_{lk}}$$

$$= d \sum_{m=0}^{\infty} \frac{x^{k+md}}{(k+md)!}$$

$$= d \cdot f_k(x)$$

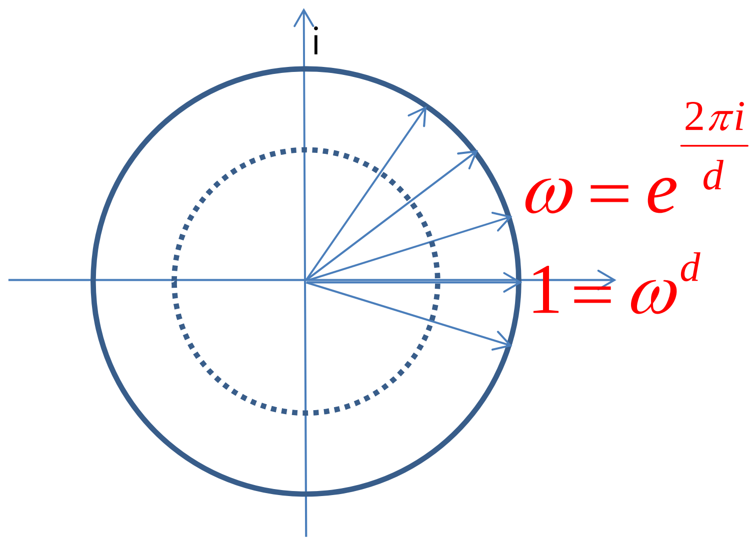
$$f_k(x) = \frac{1}{d} \sum_{s=0}^{d-1} \omega^{-ks} e^{\omega^s x}$$

$$e^{\omega^s x} = \sum_{k=0}^{d-1} \sum_{m=0}^{\infty} \frac{(\omega^s x)^{k+md}}{(k+md)!} = \sum_{k=0}^{d-1} \omega^{ks} \sum_{m=0}^{\infty} \frac{x^{k+md}}{(k+md)!} = \sum_{k=0}^{d-1} \omega^{ks} f_k(x)$$

Conjugate Relations

$$e_s(x) \equiv e^{\omega^s x} \quad \text{with} \quad \omega = e^{\frac{2\pi i}{d}}.$$

$$\begin{cases} f_k(x) = \dfrac{1}{d}\displaystyle\sum_{s=0}^{d-1} \omega^{-ks} e_s(x) \\ e_s(x) = \displaystyle\sum_{k=0}^{d-1} \omega^{ks} f_k(x) \end{cases}$$



An Interesting Observation of a Coherent State

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

$$= \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^0}{\sqrt{0!}}\right)|0\rangle + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^1}{\sqrt{1!}}\right)|1\rangle + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^2}{\sqrt{2!}}\right)|2\rangle + \cdots + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{d-1}}{\sqrt{(d-1)!}}\right)|(d-1)\rangle$$

$$+ \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^d}{\sqrt{d!}}\right)|d\rangle + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{d+1}}{\sqrt{(d+1)!}}\right)|d+1\rangle + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{d+2}}{\sqrt{(d+2)!}}\right)|d+2\rangle + \cdots + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{2d-1}}{\sqrt{(2d-1)!}}\right)|(2d-1)\rangle$$

$$+ \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{2d}}{\sqrt{(2d)!}}\right)|2d\rangle + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{2d+1}}{\sqrt{(2d+1)!}}\right)|2d+1\rangle + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{2d+2}}{\sqrt{(2d+2)!}}\right)|2d+2\rangle + \cdots + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{3d-1}}{\sqrt{(3d-1)!}}\right)|(3d-1)\rangle$$

$$+ \cdots$$

$$= \frac{|0_d\rangle}{\sqrt{d}} + \frac{|1_d\rangle}{\sqrt{d}} + \frac{|2_d\rangle}{\sqrt{d}} + \cdots + \frac{|(d-1)_d\rangle}{\sqrt{d}}$$

$$= \frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}|k_d\rangle$$

with $|k_d\rangle = \sqrt{d}\cdot e^{-\frac{|\alpha|^2}{2}}\sum_{m=0}^{\infty}\frac{\alpha^{k+md}}{\sqrt{(k+md)!}}|k+md\rangle$

$$\frac{\langle k_d|k_d\rangle}{d} = e^{-|\alpha|^2}\sum_{m=0}^{\infty}\frac{\left(|\alpha|^2\right)^{k+md}}{(k+md)!}\xrightarrow{\text{as } |\alpha|^2\to\infty}\frac{1}{d}.$$

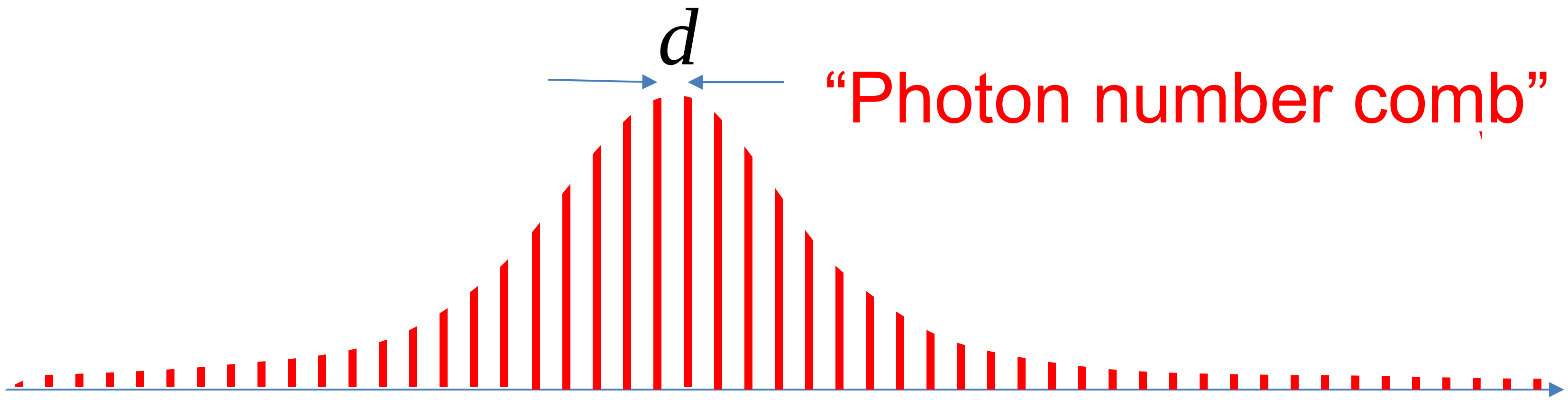
$\therefore$ As $|\alpha|^2\to\infty, \langle k_d|k_d\rangle = 1$ and $\langle k_d|l_d\rangle = \delta_{kl}$.

$$O(e^{-\frac{2\pi^2}{d^2}|\alpha|^2})$$

practically $|\alpha| \geq d$

Pseudo-number State

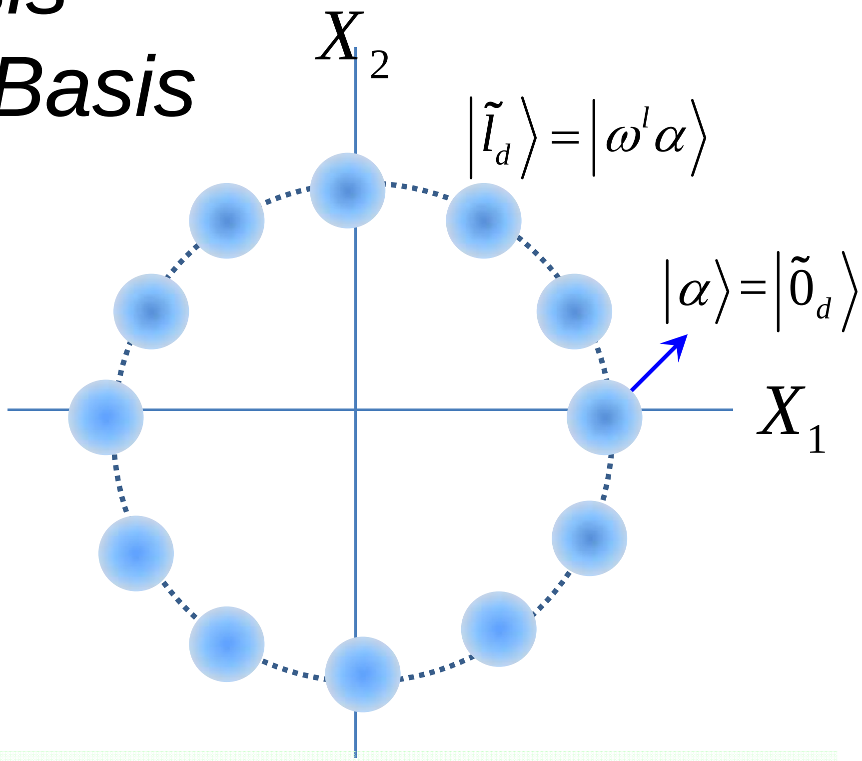
$$\left|k_d\right\rangle = \sqrt{d}\cdot e^{-\frac{|\alpha|^2}{2}}\sum_{m=0}^{\infty}\frac{\alpha^{k+md}}{\sqrt{(k+md)!}}\left|k+md\right\rangle$$



Pseudo-Number Basis and Pseudo-Phase Basis

Define $\left|\tilde{l}_d\right\rangle = \left|\omega^l \alpha\right\rangle$.

$$\begin{cases} \left|k_d\right\rangle = \dfrac{1}{\sqrt{d}} \displaystyle\sum_{l=0}^{d-1} \omega^{-kl} \left|\tilde{l}_d\right\rangle \\ \left|\tilde{l}_d\right\rangle = \dfrac{1}{\sqrt{d}} \displaystyle\sum_{l=0}^{d-1} \omega^{lk} \left|k_d\right\rangle \end{cases}$$



$\left|0_d\right\rangle$ is the 0^{th} Pseudo-number state (Computational basis) and an Even Superposition of Pseudo-Phase States (Coherent States; Conjugate Basis States).

$\left|\alpha\right\rangle = \left|\tilde{0}_d\right\rangle$ is the 0^{th} Pseudo-phase state (Conjugate basis) and an Even Superposition of Pseudo-Number States (Computational Basis States).

Orthogonal & Normalized

Exact Conjugate Relation

$$\begin{cases} \left| k_d \right\rangle = \dfrac{1}{\sqrt{d}} \sum_{l=0}^{d-1} \omega^{-kl} \left| \tilde{l}_d \right\rangle \\ \left| \tilde{l}_d \right\rangle = \dfrac{1}{\sqrt{d}} \sum_{l=0}^{d-1} \omega^{lk} \left| k_d \right\rangle \end{cases}$$

Pseudo-number states: Orthogonal, but not normalized.
Pseudo-phase states: Normalized, but not orthogonal.
As |α| gets bigger, they become orthonormalized.

$$O(e^{-\frac{2\pi^2}{d^2}|\alpha|^2}) \qquad \text{practically } |\alpha| \geq d$$

Qubits and Qu*d*its

Computational Basis

$\{|0\rangle, |1\rangle\}$

Conjugate Basis

$\{|+\rangle, |-\rangle\}$

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

Computational Basis

: pseudo-number basis

$\{|0_d\rangle, |1_d\rangle, \cdots, |(d-1)_d\rangle\}$

Conjugate Basis

: pseudo-phase basis

$\{|\tilde{0}_d\rangle = |\alpha\rangle, |\tilde{1}_d\rangle = |\omega\alpha\rangle, \cdots, |\widetilde{(d-1)}_d\rangle\}$

$$\omega = e^{\frac{2\pi i}{d}}$$

$$\begin{cases} |k_d\rangle = \dfrac{1}{\sqrt{d}} \displaystyle\sum_{s=0}^{d-1} \omega^{-kl} |\tilde{l}_d\rangle \\ |\tilde{l}_d\rangle = \dfrac{1}{\sqrt{d}} \displaystyle\sum_{k=0}^{d-1} \omega^{lk} |k_d\rangle \end{cases}$$

Qubit Operators and Qu*d*it Operators

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\text{CNOT} = X_{12} = |0\rangle_{1\,1}\langle 0| \otimes I_2 + |1\rangle_{1\,1}\langle 1| \otimes X_2$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\text{C-Z} = Z_{12} = |0\rangle_{1\,1}\langle 0| \otimes I_2 + |1\rangle_{1\,1}\langle 1| \otimes Z_2$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = |+\rangle\langle 0| + |-\rangle\langle 1|$$

$$X = \begin{bmatrix} 0 & 1 & 0 & & & \\ & 0 & 1 & \ddots & & \\ & & 0 & \ddots & \ddots & \\ & & & \ddots & 1 & 0 \\ & & & & 0 & 1 \\ 1 & & & & & 0 \end{bmatrix}$$

$$\omega = e^{\frac{2\pi i}{d}}$$

$$Z = \begin{bmatrix} 1 & 0 & 0 & \cdots & & 0 \\ & \omega & 0 & \ddots & & \\ & & \omega^2 & \ddots & \ddots & \vdots \\ & & & \ddots & 0 & 0 \\ & & & & \omega^{d-2} & 0 \\ 0 & & & & & \omega^{d-1} \end{bmatrix} = \sum_{k=0}^{d-1} \omega^k |k_d\rangle\langle k_d| = \omega^{\hat{n}}$$

$$\text{C-Z} = Z_{12} = |0_d\rangle_{1\,1}\langle 0_d| \otimes I_2 + |1_d\rangle_{1\,1}\langle 1_d| \otimes Z_2 + |2_d\rangle_{1\,1}\langle 2_d| \otimes Z^2{}_2 + \cdots$$

$$= \sum_{k=0}^{d-1} \sum_{l=0}^{d-1} |k_d\rangle_{1\,1}\langle k_d| \omega^{kl} |l_d\rangle_{2\,2}\langle l_d| = \omega^{\hat{n}_1 \hat{n}_2}$$

$$H = \frac{1}{\sqrt{d}} \begin{bmatrix} 1 & 1 & 1 & \cdots & \cdots & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \cdots & \\ 1 & \omega^2 & \omega^4 & \omega^6 & & \\ \vdots & \omega^3 & \omega^6 & \ddots & & \\ \vdots & \vdots & & & \ddots & \\ 1 & & & & & \omega^{(d-1)^2} \end{bmatrix} = \sum_{k=0}^{d-1} |\tilde{k}_d\rangle\langle k_d|$$

Cf. D. L. Zhou et al., Phys. Rev. A 68, 062303 (2003).

D. T. Pegg and S. M. Barnett,
Phys. Rev. A 39, 1665 (1989).

Generalized X Operator
Pseudo-Phase Operator
~ Pegg-Barnett Phase Operator

$$X = \sum_{l=0}^{d-1} \left|(l-1)_d\right\rangle\left\langle l_d\right| \quad \text{with} \quad \left|(-1)_d\right\rangle \equiv \left|(d-1)_d\right\rangle$$

$$Z = \sum_{l=0}^{d-1} \omega^l \left|l_d\right\rangle\left\langle l_d\right| = \omega^{\hat{n}}$$

Generalized Z Operator
Pseudo-Number Operator
~ Pegg-Barnett Number Operator
→ Phase shifter

$$H = \sum_{l=0}^{d-1} \left|\tilde{l}_d\right\rangle\left\langle l_d\right|$$

Generalized Hadamard Operator
→ One-step teleportation

$$CZ = Z_{\mathrm{ct}} = \sum_{l=0}^{d-1}\sum_{m=0}^{d-1} \omega^{lm} \left|l_d\right\rangle_{\mathrm{c\,c}}\left\langle l_d\right| \otimes \left|m_d\right\rangle_{\mathrm{c\,c}}\left\langle m_d\right| = \omega^{\hat{n}_1 \hat{n}_2}$$

Generalized Cont-Z Operator
→ Cross Kerr Interaction

Generalized Controlled-Z Operator

$H = -\chi \hat{n}_1 \hat{n}_2$ Cross Kerr Interaction

$|\chi L| = \frac{2\pi}{d}, \; d \lesssim |\alpha|$

($d \cong$ 10~1000 ?!)

$$U = e^{i\chi L \hat{n}_1 \hat{n}_2} = e^{\frac{2\pi i}{d}\hat{n}_1 \hat{n}_2} = \omega^{\hat{n}_1 \hat{n}_2} = Z_{12}$$

$$Z_{12}|\alpha\rangle_1 |\alpha\rangle_2 = \omega^{\hat{n}_1 \hat{n}_2} \frac{1}{d} \sum_{k=0}^{d-1} |k_d\rangle \sum_{l=0}^{d-1} |l_d\rangle$$

$$= \frac{1}{d} \sum_{k=0}^{d-1} \sum_{l=0}^{d-1} \omega^{kl} |k_d\rangle |l_d\rangle$$

$$= \frac{1}{\sqrt{d}} \sum_{l=0}^{d-1} |\tilde{l}_d\rangle |l_d\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |k_d\rangle |\tilde{k}_d\rangle$$

Maximal Entanglement of
Pseudo-Number State and Pseudo-Phase State

“Deterministic” Generation of a Qu*d*it Cluster State

$$\prod_{\langle p,q\rangle} \omega^{\hat{n}_p \hat{n}_q} \prod_{r \in \text{lattice}} |\alpha\rangle_r$$

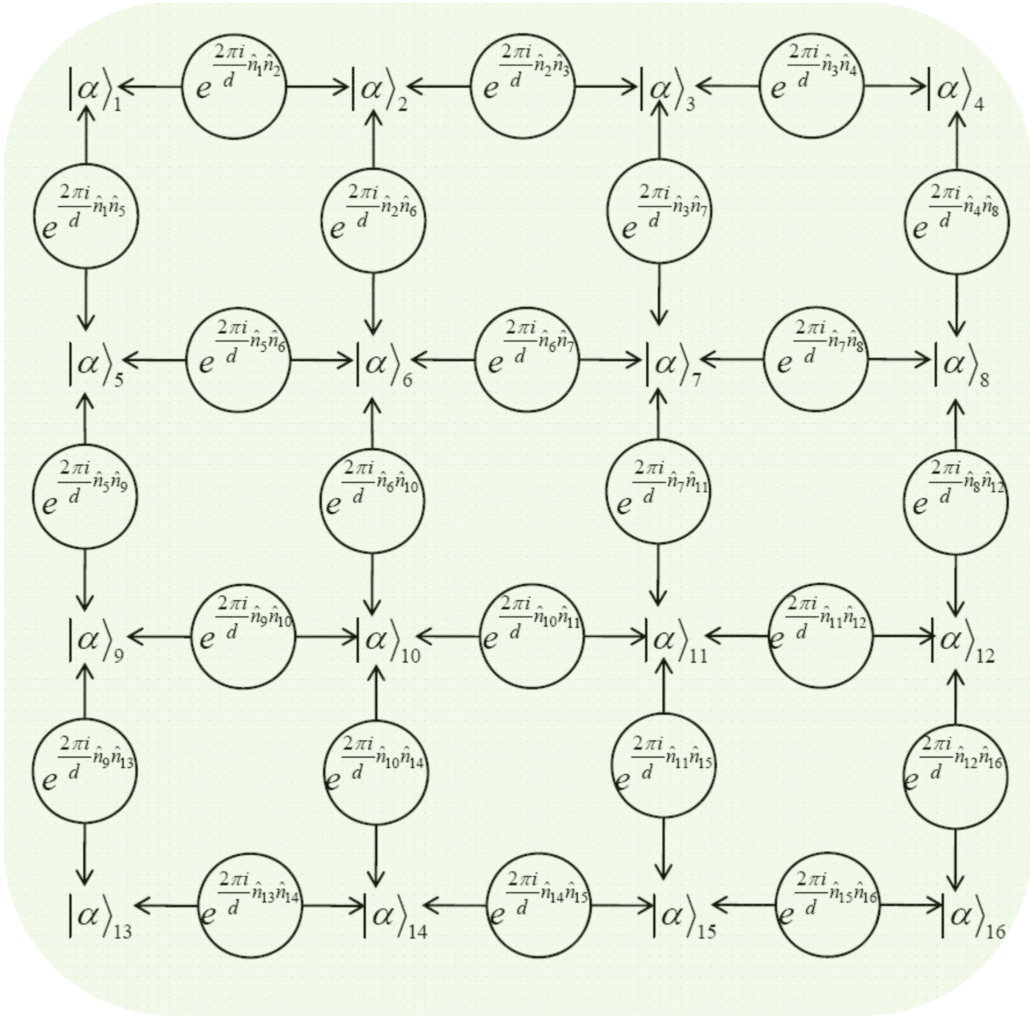
Tayloring

Scissors:

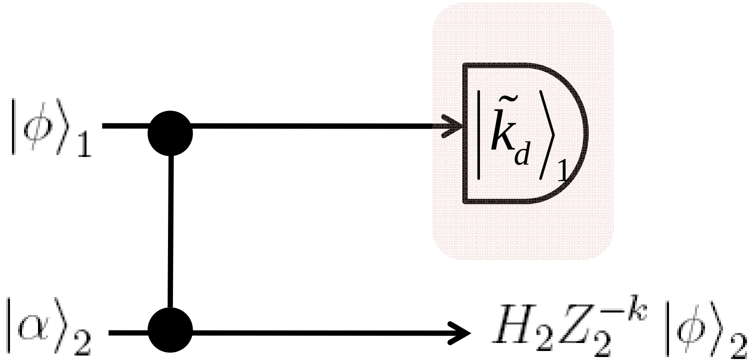
measurements in pseudo-number basis (Z)

Stitches:

measurements in pseudo-phase basis (X)



One-step
d-dim Teleportation



$$Z_{12}|\phi\rangle_1|\alpha\rangle_2 = \omega^{\hat{n}_1\hat{n}_2}\sum_{l=0}^{d-1}|l_d\rangle_1\sum_{m=0}^{d-1}\frac{|m_d\rangle_2}{\sqrt{d}}$$

$$= \sum_l\sum_m \omega^{lm} a_l |l_d\rangle_1 \frac{|m_d\rangle_2}{\sqrt{d}}$$

$$= \sum_l a_l |l_d\rangle_1 |\tilde{l}_d\rangle_2$$

$$= \sum_l a_l \left\{\sum_k \omega^{-lk}\frac{|\tilde{k}_d\rangle_1}{\sqrt{d}}\right\}|\tilde{l}_d\rangle_2$$

$$\xrightarrow[\text{into } |\tilde{k}_d\rangle_1]{\text{Projective Measurement}} \sum_l a_l \omega^{-lk}|\tilde{l}_d\rangle_2$$

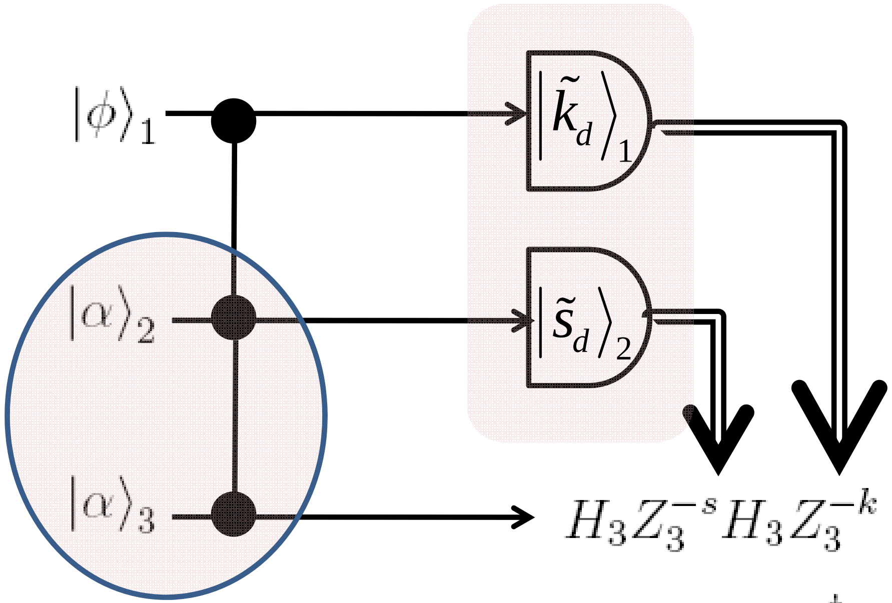
$$= \sum_l a_l \omega^{-lk} H_2 |l_d\rangle_2$$

$$= H_2 \sum_l a_l \omega^{-lk}|l_d\rangle_2$$

$$= H_2 Z_2^{-k}\sum_l a_l |l_d\rangle_2$$

$$= H_2 Z_2^{-k}|\phi\rangle_2$$

d-dim Teleportation



d-dim Bell state

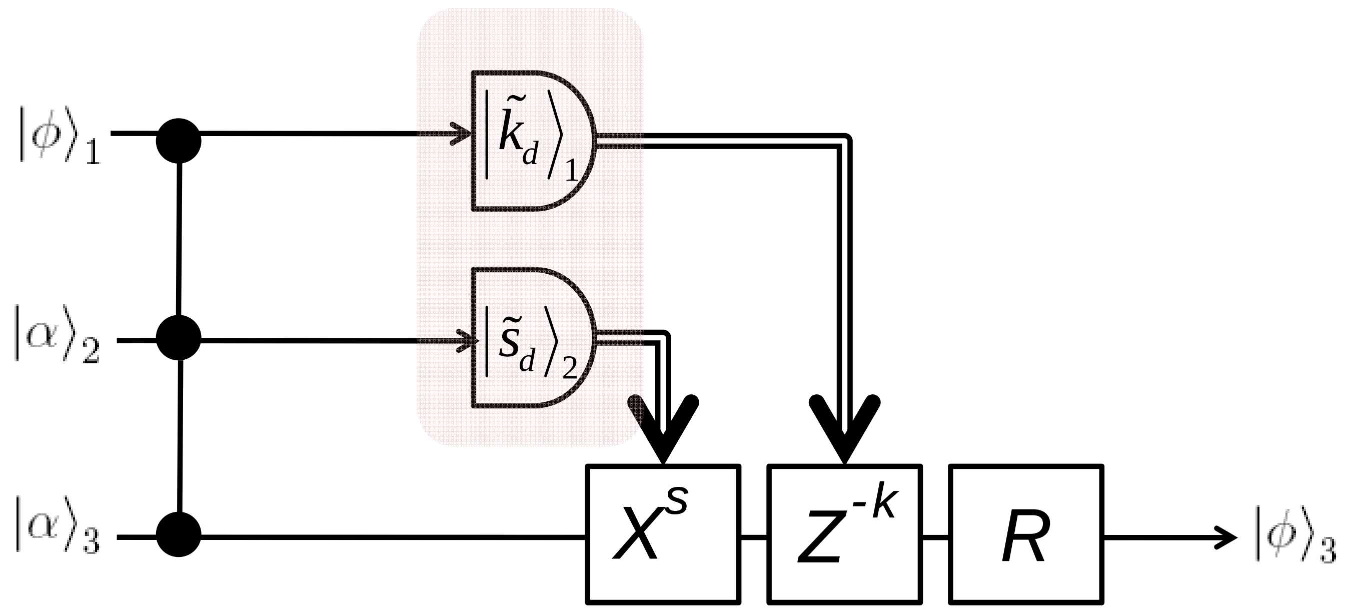
$$H_3 Z_3^{-s} H_3 Z_3^{-k} |\phi\rangle_3$$

$$= H_3 Z_3^{-s} H_3^{\dagger} H_3 H_3 Z_3^{-k} |\phi\rangle_3$$

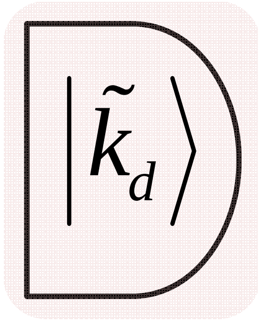
$$= X_3^{-s} R_3 Z_3^{-k} |\phi\rangle_3$$

$$= X_3^{-s} Z_3^{k} R_3 |\phi\rangle_3$$

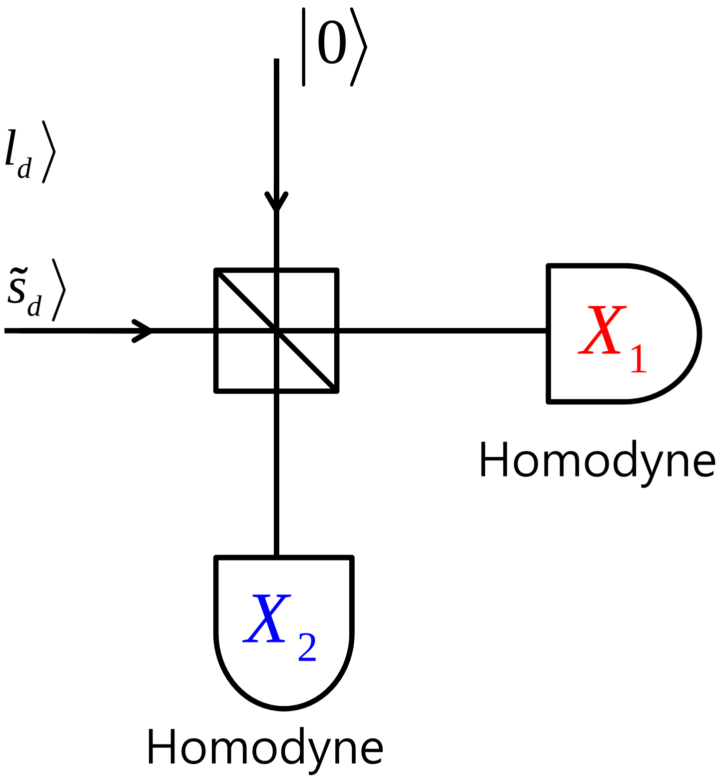
d-dim Teleportation

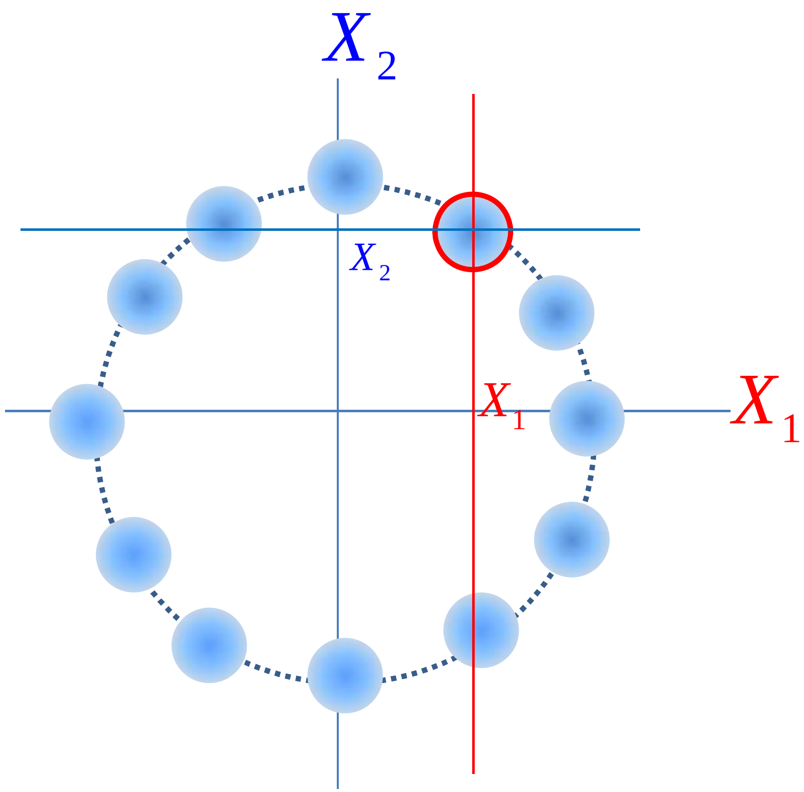


Pseudo-Phase Measurement by Homodyne Detection

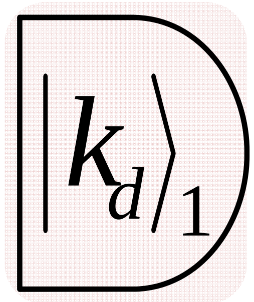


$$|\psi\rangle = \sum_l c_l |l_d\rangle = \sum_s c'_s |\tilde{s}_d\rangle$$



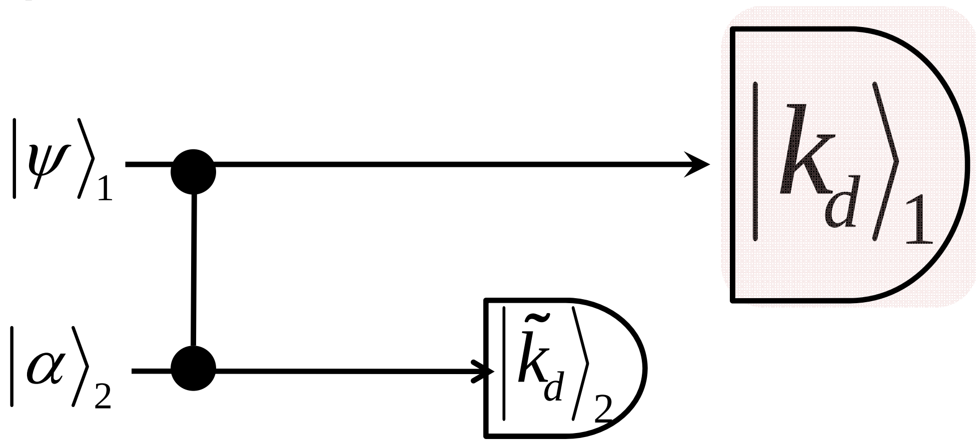


Pseudo-Number Measurement



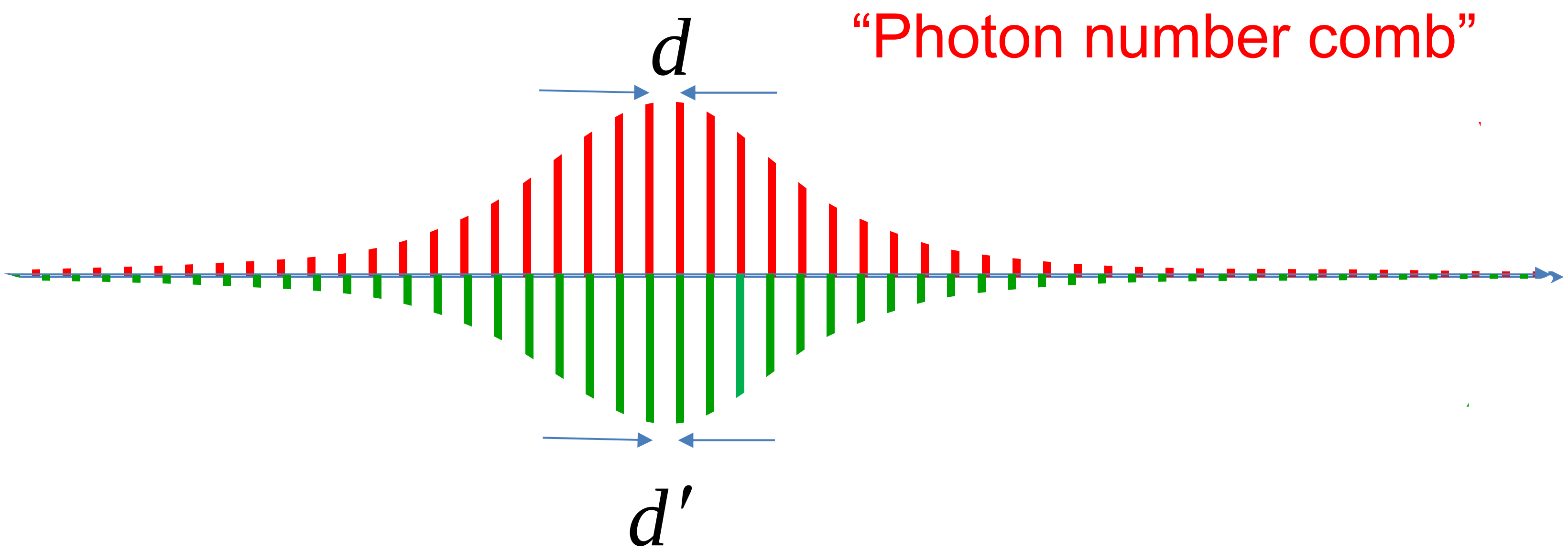
$$|\psi\rangle = \sum_l c_l |l_d\rangle = \sum_s c'_s |\tilde{s}_d\rangle$$

$$Z_{12}|\psi\rangle_1 |\alpha\rangle_2 = \sum_l c_l |l_d\rangle_1 |\tilde{l}_d\rangle_2$$

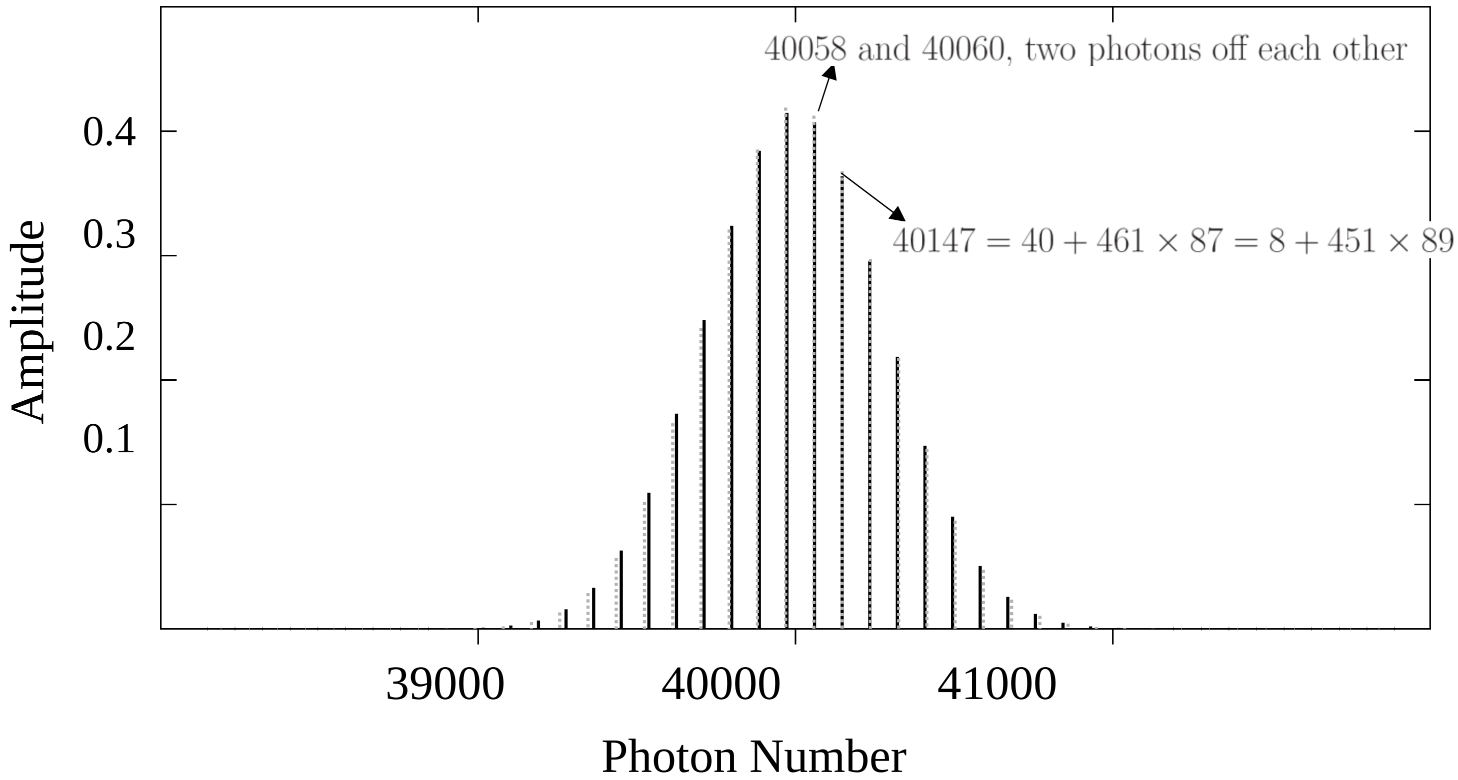


Postselection of high Number state

$$\left|k_d\right\rangle = \sqrt{d} \cdot e^{-\frac{|\alpha|^2}{2}} \sum_{m=0}^{\infty} \frac{\alpha^{k+md}}{\sqrt{(k+md)!}} \left|k+md\right\rangle$$



$$l + md = k + nd' = N$$



40058 and 40060, two photons off each other
40147 = 40 + 461 × 87 = 8 + 451 × 89
0.4
0.3
0.2
0.1
Amplitude
39000
40000
41000
Photon Number

One-step teleportation

$$\omega^{\hat{n}_1\hat{n}_2}|\phi\rangle_1|\alpha\rangle_2 = \sum_l \sum_m \omega^{lm} a_l \underset{|\widetilde{p}\rangle_1}{\left|l\right\rangle_1} \frac{\left|\underset{\sim}{m}\right\rangle_2}{\sqrt{d}}$$

$$= \sum_l a_l|\underset{\sim}{l}\rangle_1|\widetilde{l}\rangle_2$$

$$\xrightarrow{\text{measure qudit 1 into } |\widetilde{p}\rangle_1} \sum_l a_l \omega^{-pl}|\widetilde{l}\rangle_2$$

$$= \sum_l a_l H Z^{-p}|\underset{\sim}{l}\rangle_2$$

Quantum Repeater

3 qudits in series

$|\widetilde{p}\rangle_1$

$$
\begin{aligned}
\omega^{\hat{n}_0\hat{n}_1}\omega^{\hat{n}_1\hat{n}_2}|\alpha\rangle_0|\alpha\rangle_1|\alpha\rangle_2 &= \sum_l |\widetilde{l}\rangle_0|\underset{\sim}{l}\rangle_1|\widetilde{l}\rangle_2 \\
& \sum_l |\widetilde{l}\rangle_0 \omega^{-pl}|\widetilde{l}\rangle_2 \\
&= \sum_l |\widetilde{l}\rangle_0 H Z^{-p}|\underset{\sim}{l}\rangle_2,
\end{aligned}
$$

Quantum Repeater

4 qudits in series

$$|\widetilde{p}\rangle_1 \quad |\widetilde{q}\rangle_2$$

$$\omega^{\hat{n}_0\hat{n}_1}\omega^{\hat{n}_1\hat{n}_2}\omega^{\hat{n}_2\hat{n}_3}|\alpha\rangle_0|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3$$

$$\xrightarrow[\text{measure qudit 2 into } |\widetilde{q}\rangle_2]{\text{measure qudit 1 into } |\widetilde{p}\rangle_1} \sum_l |\widetilde{l}\rangle_0 H Z^{-q} H Z^{-p} |l\rangle_3$$

$$= \sum_l |\widetilde{l}\rangle_0 H Z^{-q} \omega^{-pl} |\widetilde{l}\rangle_3$$

$$= \sum_l |\widetilde{l}\rangle_0 \omega^{-pl} |q - l\rangle_3$$

Quantum Repeater

5 qudits in series

$$|\widetilde{p}\rangle_1 \quad |\widetilde{q}\rangle_2 \quad |\widetilde{r}\rangle_3$$

$$\omega^{\hat{n}_0\hat{n}_1}\omega^{\hat{n}_1\hat{n}_2}\omega^{\hat{n}_2\hat{n}_3}\omega^{\hat{n}_3\hat{n}_4}|\alpha\rangle_0|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4$$

$$\begin{aligned}
& \sum_l |\widetilde{l}\rangle_0 HZ^{-r}HZ^{-q}HZ^{-p}|\underset{\sim}{l}\rangle_4 \\
= & \sum_l |\widetilde{l}\rangle_0 HZ^{-r}HZ^{-q}\omega^{-pl}|\widetilde{l}\rangle_4 \\
= & \sum_l |\widetilde{l}\rangle_0 HZ^{-r}\omega^{-pl}|\underset{\sim}{q-l}\rangle_4 \\
= & \sum_l |\widetilde{l}\rangle_0 \omega^{-pl}\omega^{-r(q-l)}|\widetilde{q-l}\rangle_4
\end{aligned}$$

Bell State

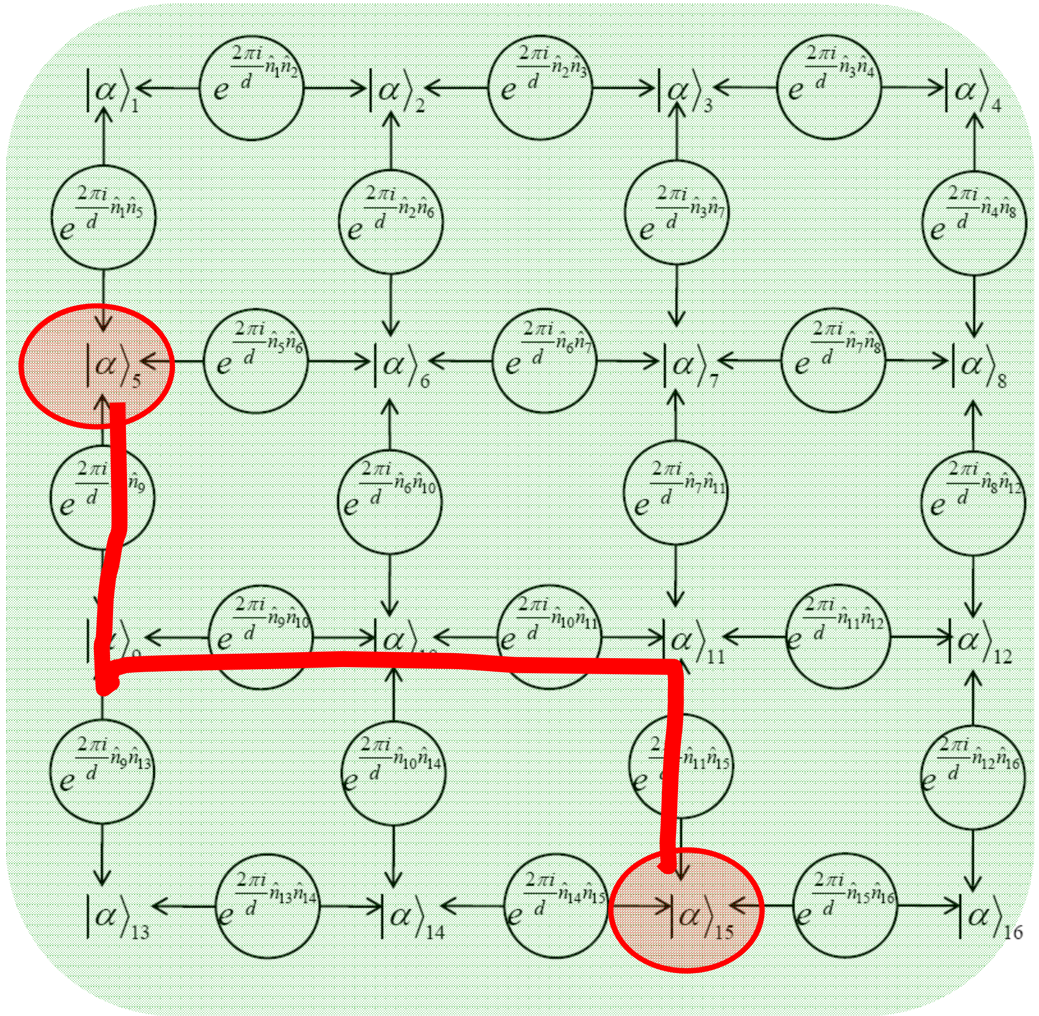
Tayloring

Scissors:

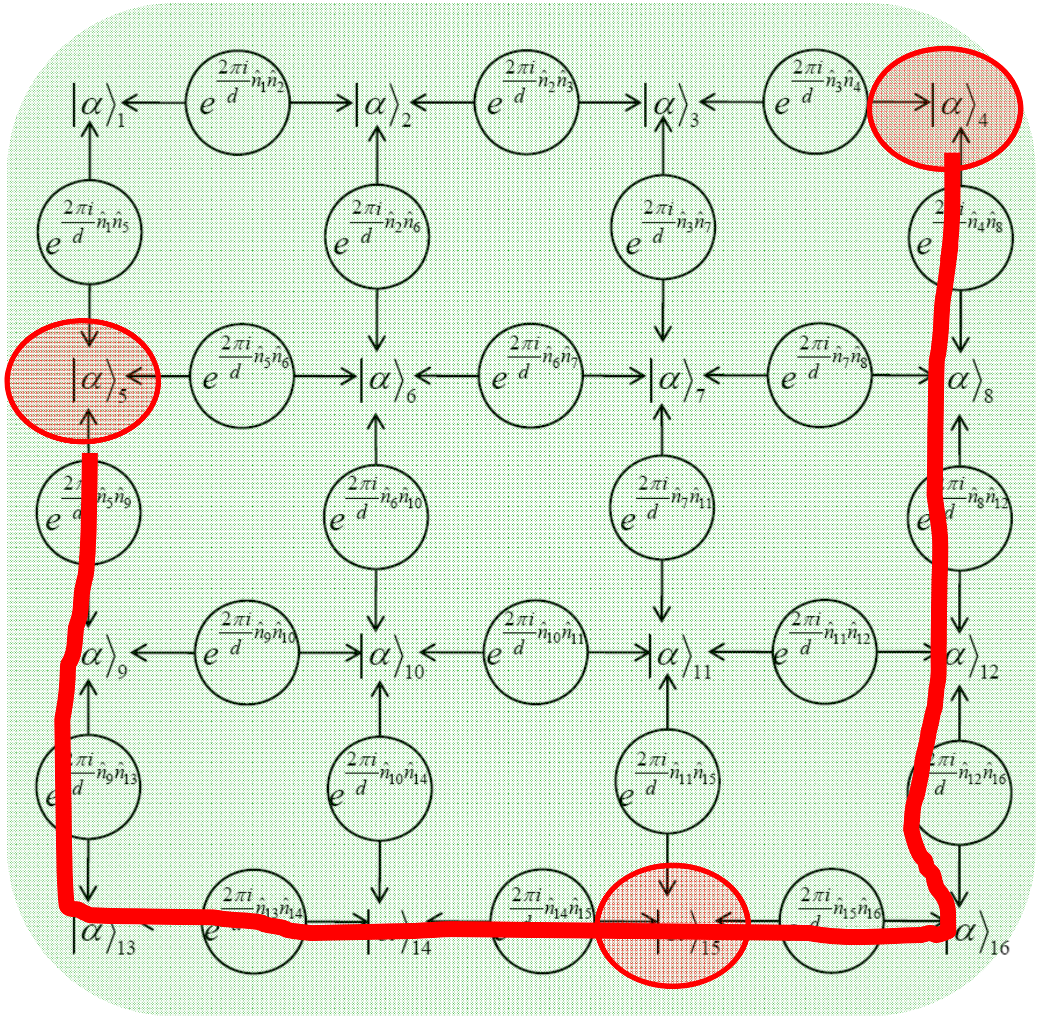
measurements in pseudo-number basis (Z)

Stitches:

measurements in pseudo-phase basis (X)



GHZ State



***Taylorin*g**

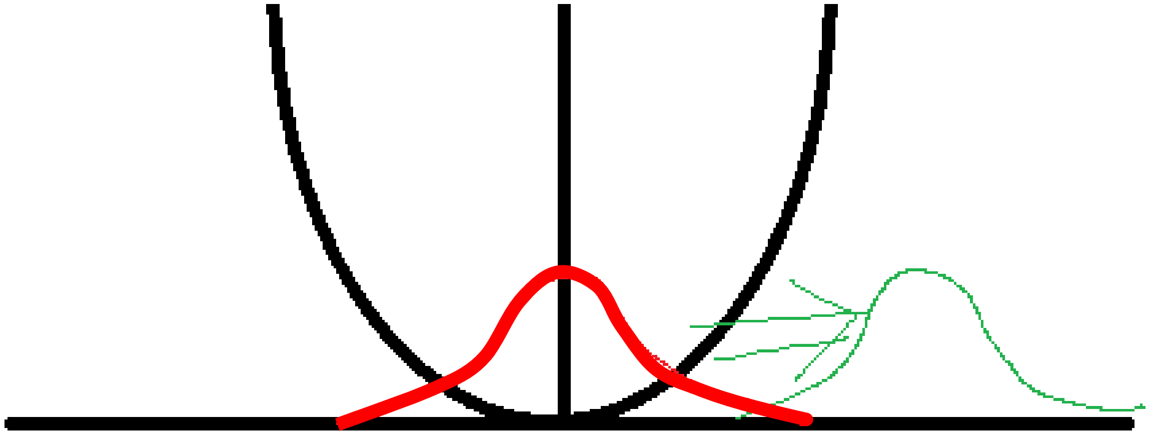
Scissors:

measurements in pseudo-number basis (Z)

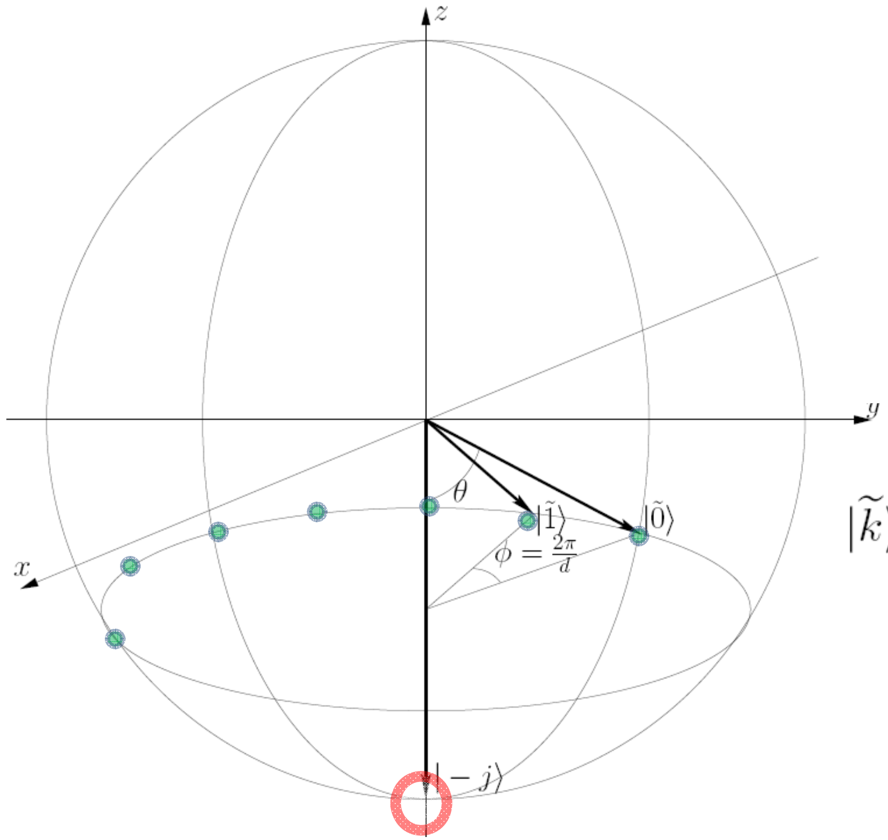
Stitches:

measurements in pseudo-phase basis (X)

Coherent State



Spin Coherent State *Qudit*



$$e^{-i\theta J_y}|-j\rangle = |\widetilde{0}\rangle = \sum_{\lambda=-j}^{j} |\lambda\rangle\langle\lambda|e^{-i\theta J_y}|-j\rangle$$

$$|\widetilde{k}\rangle = e^{-i\phi J_z k} e^{-i\theta J_y}|-j\rangle \text{ with } k = 0, \cdots, d-1.$$

$$|\langle\widetilde{0}|\widetilde{1}\rangle| = \left|\frac{1 + \hat{n}_{|\widetilde{0}\rangle} \cdot \hat{n}_{|\widetilde{1}\rangle}}{2}\right|^j \approx e^{-j\frac{\pi^2}{d^2}\sin^2\theta}$$

Modulo-*d* spin state & Spin coherent state

$$|\underset{\sim}{l}\rangle = \sqrt{d} \sum_{m}^{-j\leq l+md\leq j} |l+md\rangle\langle l+md|e^{-i\theta J_y}|-j\rangle,$$

$$|\widetilde{k}\rangle = \frac{1}{\sqrt{d}} \sum_{l=0}^{d-1} \omega^{-kl} |\underset{\sim}{l}\rangle$$

$$|\underset{\sim}{l}\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} \omega^{lk} |\widetilde{k}\rangle$$

Ising Interaction→CZ

$$
\begin{aligned}
& e^{-\frac{2\pi i}{d} J_{z1} J_{z2}} |\widetilde{0}\rangle_1 |\widetilde{0}\rangle_2 \\
= \quad & \frac{1}{d} \sum_{k=0}^{d-1} \sum_{l=0}^{d-1} \omega^{-kl} |\underset{\sim}{k}\rangle_1 |\underset{\sim}{l}\rangle_2 \\
= \quad & \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} \left|\widetilde{k}\right\rangle_1 \left|\underset{\sim}{k}\right\rangle_2 \text{ or } \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} \left|\underset{\sim}{k}\right\rangle_1 \left|\widetilde{k}\right\rangle_2
\end{aligned}
$$

Summary

JK, J. Lee, S.-W. Ji, H. Nha, P. Anisimov, J. P. Dowling, Opt Comm 337, 79 (2015)

- Optical Coherent State: Even superposition of *d*-dim pseudo-number computational basis states
- Generalized Cont-Z can be implemented by Cross-Kerr interaction ($d \cong$ 10~1000 ?!)
  → Max Entanglement ➔ Qu***d***it Cluster State
- *d*-dim teleportation
- Pseudo-Phase Measurement by Homodyne detection
- Pseudo-Number Measurement
- Network for Quantum Communication
- Spin coherent state qudit
- Qu*d*it Cluster Quantum Computation …

Decoherence

Single qu*d*it operation with non-integer power