Dynamical maps and memory kernels

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Mathematical Aspects of Quantum Information
NIMS, Daejeon, February 2016


APTEKA

Quantum evolution

$\dim \mathcal{H} = n$; $\mathfrak{S}(\mathcal{H})$ – space of quantum states (density matrices)

$$\Lambda_t : \mathfrak{S}(\mathcal{H}) \longrightarrow \mathfrak{S}(\mathcal{H})$$

$$\rho_t = \Lambda_t[\rho] \ ; \quad t \geq 0$$

- Λ_t is completely positive and trace-preserving (CPTP)
- $\Lambda_0 = \mathbb{1}$

(quantum) dynamical map

Physics likes equations of motions

We look for the linear equation for Λ_t

Basic example — unitary evolution

$$H = H^*$$

$$L[\rho] = -i[H, \rho]$$

$$\frac{d}{dt}\Lambda_t = L\Lambda_t \ ; \quad \Lambda_0 = \mathbb{1}$$

$$\Lambda_t[\rho] = U_t \rho U_t^* \quad ; \quad U_t = e^{-iHt} \ ; \ t \in \mathbb{R}$$

Markovian semigroup

$$\frac{d}{dt}\Lambda_t = L\Lambda_t \; ; \quad \Lambda_0 = \mathbb{1} \quad \longrightarrow \quad \Lambda_t = e^{tL} \; ; \; t \geq 0$$

What is the most general L ?

Theorem (GKSL (1976))

$\Lambda_t = e^{tL}$ *is CPTP iff*

$$L[\rho] = -i[H,\rho] + \left(\Phi[\rho] - \frac{1}{2}\{\Phi^*[\mathbb{I}],\rho\} \right)$$

$H^* = H$; Φ – *arbitrary CP*

Quantum Open Systems

$$S + E \quad \longrightarrow \quad \mathcal{H}_S \otimes \mathcal{H}_E$$

$$U_t = e^{-iHt}$$

$$t = 0 \to \rho \otimes \omega$$

$$\rho_t := \Lambda_t[\rho] = \mathrm{Tr}_E\left[U_t \rho \otimes \omega U_t^\dagger\right]$$

Nakajima-Zwanzig equation

$$\rho_t := \Lambda_t[\rho] = \mathrm{Tr}_E\left[U_t \rho \otimes \omega U_t^\dagger\right]$$

$$\frac{d}{dt}\Lambda_t = \int_0^t K_{t-\tau}\Lambda_\tau \, d\tau$$

One obtains a Markovian semigroup e^{tL} only in the very special regime $K(t) \longrightarrow \delta(t)L$

$$\frac{d}{dt}\Lambda_t = L\Lambda_t$$

Local vs. non-local

- non-local master equation (Nakajima-Zwanzig equation)

$$\frac{d}{dt}\Lambda_t = \int_0^t K_{t-\tau}\Lambda_\tau \, d\tau$$

- local in time master equation

$$\frac{d}{dt}\Lambda_t = L_t\Lambda_t$$

$$\Lambda_t \quad \longrightarrow \quad L_t = \dot{\Lambda}_t\Lambda_t^{-1}$$

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local generator

$$L_t[\rho] \;=\; -i[H_t,\rho] + \left(\Phi_t[\rho] - \frac{1}{2}\{\Phi_t^*[\mathbb{I}],\rho\}\right)$$

non-local (memory kernel) generator

$$K_t[\rho] \;=\; -i[G_t,\rho] + \left(\Psi_t[\rho] - \frac{1}{2}\{\Psi_t^*[\mathbb{I}],\rho\}\right)$$

$H_t\,, G_t$ – hermitian ; $\Phi_t\,, \Psi_t$ – hermiticity-preserving

These structures guarantee that Λ_t is hermiticity- and trace-preserving

What about complete positivity?

What is known:

$$L_t[\rho] \;=\; -i[H_t,\rho] + \left(\Phi_t[\rho] - \frac{1}{2}\{\Phi_t^*[\mathbb{I}],\rho\}\right)$$

$$\Lambda_t = \mathcal{T}\exp\left(\int_0^t L_\tau d\tau\right)$$

$$\Phi_t \text{ –CP} \implies \Lambda_t \text{ –CPTP}$$

What is known:

$$K_t[\rho] \;=\; -i[G_t,\rho] + \left(\Psi_t[\rho] - \frac{1}{2}\{\Psi_t^*[\mathbb{I}],\rho\}\right)$$

NO general results were known!

only so-called semi-Markov classical evolution was characterized

I provide sufficient conditions during this talk

Example: qubit dephasing

$$L_t[\rho] = \frac{1}{2}\gamma(t)[\sigma_3\rho\sigma_3 - \rho]$$

$$\Lambda_t[\rho] = \begin{pmatrix} \rho_{11} & \lambda(t)\rho_{12} \\ \lambda(t)\rho_{21} & \rho_{22} \end{pmatrix}$$

$$CP \iff |\lambda(t)| \leq 1$$

$$\dot{\lambda}(t) = -\gamma(t)\lambda(t) \ \ ; \ \ \lambda(0) = 1 \ \longrightarrow \ \lambda(t) = \exp\left(-\int_0^t \gamma(\tau)d\tau\right)$$

Example: qubit dephasing

$$K_t[\rho] = \frac{1}{2} k(t) [\sigma_3 \rho \sigma_3 - \rho]$$

$$\dot{\lambda}(t) = -\int_0^t k(t-\tau)\lambda(\tau) d\tau \ \ ; \ \ \lambda(0) = 1 \quad \longrightarrow \quad \lambda(t) = ???$$

Laplace: $\lambda(t) \rightarrow \widetilde{\lambda}(s) = \int_0^\infty e^{-st}\lambda(t)dt$

$$\dot{\lambda}(t) = -\int_0^t k(t-\tau)\lambda(\tau)d\tau \quad ; \quad \lambda(0) = 1$$

$$s\widetilde{\lambda}(s) - 1 = -\widetilde{k}(s)\widetilde{\lambda}(s) \quad \longrightarrow \quad \widetilde{\lambda}(s) = \frac{1}{s + \widetilde{k}(s)}$$

$$\widetilde{\lambda}(s) \longrightarrow \lambda(t)$$

CP $\iff$ $\lambda(t) \in [-1, 1]$

NO simple condition for $k(t)$

$$\Lambda_t[\rho] = \begin{pmatrix} \rho_{11} & \lambda(t)\rho_{12} \\ \lambda(t)\rho_{21} & \rho_{22} \end{pmatrix}$$

If $\gamma(t)$ satisfies $\Gamma(t) := \int_0^t \gamma(\tau)d\tau < \infty$ for finite t

time-local $\longrightarrow$ $\lambda(t) = e^{-\Gamma(t)} \geq 0$

Λ_t is CP $\iff$ $\lambda(t) \in [-1, 1]$

$\lambda(t) \not\geq 0 \implies \gamma(t)$ is singular

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Example: let $\lambda(t) = \cos t$; (non-Markovian evolution – next talk)

$$\Lambda_t[\rho] = \begin{pmatrix} \rho_{11} & \rho_{12}\cos t \\ \rho_{21}\cos t & \rho_{22} \end{pmatrix}$$

$$\gamma(t) = \tan t \qquad \text{(singular)}$$

$$k(t) = 1 \quad (t \geq 0) \qquad \text{(regular)}$$

non-trivial & singular L_t $\longleftrightarrow$ simple & regular K_t

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Laplace transform domain

$$\frac{d}{dt}\Lambda_t = \int_0^t K_{t-\tau}\Lambda_\tau \, d\tau$$

$$s\widetilde{\Lambda}_s - \mathbb{1} = \widetilde{K}_s\widetilde{\Lambda}_s$$

$$\widetilde{\Lambda}_s = \frac{1}{s - \widetilde{K}_s} \quad \longrightarrow \quad \Lambda_t = ???$$

semigroup $\longrightarrow \widetilde{\Lambda}_s = \frac{1}{s - L} \quad \longrightarrow \quad \Lambda_t = e^{tL}$

- Λ_t CP
- what about $\widetilde{\Lambda}_s$?

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- Λ_t CP
- what about $\widetilde{\Lambda}_s$?

Completely monotone functions

A function $f: [0, \infty) \to R$ is completely monotone (CM) if

$$(-1)^n \frac{d^n}{ds^n} f(s) \geq 0, \quad n = 0, 1, 2, \ldots$$

Theorem (Bernstein)

A function $f: [0, \infty) \to R$ is CM iff it is a Laplace transform of a positive function $g(t) \geq 0$

$$f(s) = \int_0^\infty e^{-st} g(t) dt$$

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Completely monotone functions — examples

$$f(t) = \frac{1}{t+a}$$

$$f(t) = e^{-at} \quad ; \quad a > 0$$

If f and g are CM, then $f \cdot g$ – CM

Completely monotone functions — examples

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"Quantum Bernstein theorem"

$$\frac{d}{dt}\Lambda_t = \int_0^t K_{t-\tau}\Lambda_\tau \, d\tau$$

Λ_t — CP

$$s\widetilde{\Lambda}_s - \mathbb{1} = \widetilde{K}_s\widetilde{\Lambda}_s$$

$(-1)^n \frac{d^n}{ds^n}\widetilde{\Lambda}_s$ — CP $\quad n = 0, 1, 2, \ldots$ CM-CP

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How to construct a legitimate K_t ?

Markovian semigroup — quantum jumps representation

$$L[\rho] = -i[H,\rho] + \left(B[\rho] - \frac{1}{2}\{B^*[\mathbb{I}],\rho\} \right)$$

$$X = \frac{1}{2} B^*[\mathbb{I}] \geq 0$$

$$L[\rho] = B[\rho] - [i(H\rho - \rho H) + (X\rho + \rho X)]$$

$$L = B - Z$$

$$Z[\rho] = i(C\rho - \rho C^\dagger) \;\; ; \;\; C = H - \frac{i}{2} X$$

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$$\dot{\Lambda}_t = L\Lambda_t \quad ; \quad \Lambda_0 = \mathbb{1}$$

$$\dot{N}_t = -ZN_t \quad ; \quad N_0 = \mathbb{1} \quad \longrightarrow \quad N_t[\rho] = e^{-iCt}\rho e^{iC^\dagger t}$$

$$\widetilde{N}_s = \frac{1}{s+Z} \quad ; \quad \widetilde{\Lambda}_s = \frac{1}{s-B+Z}$$

$$\widetilde{\Lambda}_s = \widetilde{N}_s + \widetilde{N}_s B \widetilde{\Lambda}_s$$

$$\widetilde{\Lambda}_s = \widetilde{N}_s(\mathbb{1} + \widetilde{Q}_s + \widetilde{Q}_s\widetilde{Q}_s + \ldots) = \widetilde{N}_s \frac{1}{1-\widetilde{Q}_s}$$

$$Q_t := BN_t$$

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$$\Lambda_t = N_t + N_t * (Q_t + Q_t * Q_t + Q_t * Q_t * Q_t + \ldots)$$

$$N_t, Q_t = BN_t \text{ –CP}$$

quantum jump representation

$$\begin{aligned} \Lambda_t &= N_t + \int_0^t dt_1 N_{t-t_1} B N_{t_1} + \ldots \\ &= N_t + N_t * BN_t + N_t * BN_t * BN_t + \ldots \end{aligned}$$

$L \longrightarrow K_t$

$$L = B - Z \;\; \rightarrow \;\; \dot{N}_t = -Z N_t \;\; ; \;\; N_0 = \mathbb{1} \;\; \longrightarrow \;\; N_t[\rho] = e^{-iCt} \rho e^{iC^\dagger t}$$

$$\Lambda_t = N_t + N_t * (Q_t + Q_t * Q_t + Q_t * Q_t * Q_t + \dots) \; ; \quad Q_t := B N_t$$

$$K_t = B_t - Z_t \;\; \rightarrow \;\; \dot{N}_t = -\int_0^t Z_{t-\tau} N_\tau d\tau \;\; ; \;\; N_0 = \mathbb{1} \;\; \longrightarrow \;\; N_t \text{ – CP}$$

$$\Lambda_t = N_t + N_t * (Q_t + Q_t * Q_t + Q_t * Q_t * Q_t + \dots) \; ; \quad Q_t := B_t * N_t \text{ – CP}$$

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$$\Lambda_t = N_t + N_t * (Q_t + Q_t * Q_t + Q_t * Q_t * Q_t + \ldots) \;; \quad Q_t := B_t * N_t \text{ – CP}$$

We call $\{N_t, Q_t\}$ a legitimate pair iff

- N_t, Q_t are CP, and $N_0 = \mathbb{1}$
- $\mathrm{Tr}[(Q_t + \dot{N}_t)\rho] = 0$,
- $||\widetilde{Q}_s|| \leq 1$

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Example:

$$N_t = \left(1 - \int_0^t f(\tau)d\tau\right) \mathbb{1}$$

$$f(t) \geq 0 \ \ ; \ \ \int_0^\infty f(\tau)d\tau \leq 1$$

$$Q_t = f(t)B \ ; \quad B \text{ –CPTP} \implies ||\widetilde{Q}_s||_1 = |\widetilde{f}(s)| \leq 1$$

$$K_t = k(t)(B - \mathbb{1})$$

$$\widetilde{k}(s) = \frac{s\widetilde{f}(s)}{1 - f(s)}$$

$$f(t) = \gamma e^{-\gamma t} \longrightarrow \widetilde{f}(s) = \frac{\gamma}{s+\gamma} \longrightarrow \widetilde{k}(s) = \gamma \longrightarrow k(t) = \gamma\delta(t)$$

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Qubit dephasing

$$K_t = k(t)(B - \mathbb{1})$$

$$B[\rho] = \sigma_3 \rho \sigma_3$$

$$\widetilde{k}(s) = \frac{s\widetilde{f}(s)}{1 - f(s)}$$

$$f(t) \geq 0 \quad ; \quad \int_0^\infty f(\tau) d\tau \leq 1$$

$$K_t = k(t)(B - \mathbb{1})$$

$$B^2 = B \quad \text{–CPTP projector}$$

$$\Lambda_t = \left(1 - \int_0^t f(\tau) d\tau\right) \mathbb{1} + \int_0^t f(\tau) d\tau \, B$$

$$0 \leq \int_0^t f(\tau) d\tau \leq 1$$

but $f(t)$ needs not be positive!

Properties — 1

Convexity

If $\{N_t^{(k)}, Q_t^{(k)}\}$ are legitimate pairs then a convex combination

$$N_t = \sum_k p_k N_t^{(k)} \quad ; \quad Q_t = \sum_k p_k Q_t^{(k)}$$

provide a legitimate pair.

Properties — 2

Reduced pair

Suppose that $\{\mathbf{N}_t, \mathbf{Q}_t\}$ defines a legitimate pair for the evolution in $\mathcal{H} \otimes \mathcal{H}_E$. Then for arbitrary state ω in $\mathcal{H}_E$

$$N_t[\rho] = \mathrm{Tr}_E(\mathbf{N}_t[\rho \otimes \omega]), \ Q_t[\rho] = \mathrm{Tr}_E(\mathbf{Q}_t[\rho \otimes \omega]),$$

provide a legitimate pair for the reduced evolution in $\mathcal{H}$.

Properties — 3

Gauge transformations

If $\{N_t, Q_t\}$ is a legitimate pair and $\mathcal{F}_t$ is a dynamical map, then

$$N_t' = \mathcal{F}_t N_t \ ; \quad Q_t' = \mathcal{F}_t Q_t,$$

provide a legitimate pair as well.

Properties — 4

CP shift

If $\{N_t, Q_t\}$ is a legitimate pair and $\mathcal{G}_t$ is a linear map such that $\int_0^t \mathcal{G}_\tau d\tau$ is CP, then

$$N_t' = N_t + \int_0^t \mathcal{G}_\tau d\tau \ ; \ \ Q_t' = Q_t - \mathcal{G}_t,$$

define a legitimate pair provided Q_t' is CP.

$$N_t + \int_0^t Q_\tau d\tau \ \text{–CPTP}$$

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CP shift

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$$N_t + \int_0^t Q_\tau d\tau \ \ \text{–CPTP}$$

New equation — collision models

$$\frac{d}{dt}\Lambda_t = \int_0^t K_{t-\tau}\Lambda_\tau \, d\tau$$

$$\{N_t, Q_t\}$$

$$\frac{d}{dt}\Lambda_t = \int_0^t \mathbb{K}_{t-\tau}\Lambda_\tau \, d\tau + \frac{d}{dt}N_t$$

$$\widetilde{\mathbb{K}}_s = s\widetilde{N}_s\widetilde{Q}_s\widetilde{N}_s^{-1}$$

Summary

- I provided a relation between time-local and non-local approaches
- I provided a construction for a family of legitimate kernels in terms of legitimate pairs
- all known examples fit this class
- this class defines a natural generalization of classical semi-Markov evolution
- the necessary condition is still missing
- see the next talk for INTERESTING results on non-Markovian quantum evolution!

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