CENTRALIZING CENTRALIZERS

A.E. GUTERMAN
LOMONOSOV MOSCOW STATE UNIVERSITY (RUSSIA)

The talk is based on the joint work with G. Dolinar, B. Kuzma and P. Oblak.

For a matrix $A \in M_n(\mathbb{F})$ its centralizer

$$\mathcal{C}(A) = \{X \in M_n(\mathbb{F}) |\ AX = XA\}$$

is the set of all matrices commuting with A. For a set $S \subseteq M_n(\mathbb{F})$ its centralizer

$$\mathcal{C}(S) = \{X \in M_n(\mathbb{F}) |\ AX = XA \text{ for every } A \in S\} = \bigcap_{A \in S} \mathcal{C}(A)$$

is the intersection of centralizers of all its elements. Centralizers are important and useful both in fundamental and applied sciences.

A non-scalar matrix $A \in M_n(\mathbb{F})$ is *minimal* if for every $X \in M_n(\mathbb{F})$ with $\mathcal{C}(A) \supseteq \mathcal{C}(X)$ it follows that $\mathcal{C}(A) = \mathcal{C}(X)$. A non-scalar matrix $A \in M_n(\mathbb{F})$ is *maximal* if for every non-scalar $X \in M_n(\mathbb{F})$ with $\mathcal{C}(A) \subseteq \mathcal{C}(X)$ it follows that $\mathcal{C}(A) = \mathcal{C}(X)$.

We investigate and characterize minimal and maximal matrices over arbitrary fields.

Our results are applied to the theory of commuting graphs of matrix rings, to the preserver problems, namely to characterize commutativity preserving maps on matrices, and to the centralizers of high orders.