S^1-boundedness of triple operator integrals

Clément Coine
University of Franche-Comté

Let $\mathcal{H}$ be a separable Hilbert space. Denote by $S^2(\mathcal{H})$ the Hilbert-Schmidt class and by $S^1(\mathcal{H})$ the trace class on $\mathcal{H}$. Let A, B, C be normal operators on $\mathcal{H}$ and let $\lambda_A, \lambda_B, \lambda_C$ be scalar valued spectral measures for A, B and C, respectively. For $\phi \in L^\infty(\lambda_A \times \lambda_B)$ and $\psi \in L^\infty(\lambda_A \times \lambda_B \times \lambda_C)$, one may define a double operator integral mapping

$$\Gamma^{A,B}(\phi) : S^2(\mathcal{H}) \mapsto S^2(\mathcal{H})$$

and a triple operator integral mapping

$$\Gamma^{A,B,C}(\psi) : S^2(\mathcal{H}) \times S^2(\mathcal{H}) \mapsto S^2(\mathcal{H}).$$

When $\mathcal{H}$ is finite dimensional, $\Gamma^{A,B}(\phi)$ and $\Gamma^{A,B,C}(\psi)$ correspond to Schur multipliers and bilinear Schur multipliers with respect to the spectral decompositions of A, B and C.
In 1985, V. Peller characterized the functions ϕ for which $\Gamma^{A,B}(\phi)$ defines a bounded operator from $S^1(\mathcal{H})$ into itself. Our main result is a characterization of the functions $\psi \in L^\infty(\lambda_A \times \lambda_B \times \lambda_C)$ such that $\Gamma^{A,B,C}(\psi)$ is valued in $S^1(\mathcal{H})$. We showed that this holds true if and only if there exist a Hilbert space $\mathcal{K}$ and two functions $a \in L^\infty(\lambda_A \times \lambda_B, K)$ and $b \in L^\infty(\lambda_B \times \lambda_C, K)$ such that

$$\psi(\omega_1, \omega_2, \omega_3) = \langle a(\omega_1, \omega_2), b(\omega_2, \omega_3)\rangle$$

almost everywhere. This is a bilinear analogue of Peller's result.
This is a joint work with Christian Le Merdy (University of Franche-Comté) and Fedor Sukochev (UNSW, Sydney).