Fourier analysis arising from type III representations of the noncommutative 2-torus

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For the noncommutative 2-torus $\mathbb{A}_\alpha$, we have recently shown (FF and L. Suriano) that it is possible to construct explicitly type III representations π, at least when α is Liouville number. In addition, if α is Liouville, with a faster approximation property (*i.e.* a so called Ultra-Liouville number), it is possible to construct also genuine (*i.e.* non trivial) modular spectral triples.

For such examples (and also for any representation of type II_1 and II_∞ when the modular group is non trivial), we introduce and investigate a one-parameter family with parameter $t \in [0,1]$, of Fourier transforms arising from several embeddings of

$$L^\infty\big(\pi(\mathbb{A}_\alpha)''\big) \equiv L^\infty(M) \equiv M \hookrightarrow M_* \equiv L^1(M) \equiv L^1\big(\pi(\mathbb{A}_\alpha)''\big)\,,$$

due the non-triviality of the modular structure (where $t = 0, 1$ correspond to the left and right embedding, respectively).

For such noncommutative examples of Fourier transforms, we prove the analogous of Riemann-Lebesgue Lemma and Hausdorff-Young Theorem.

In addition, for $p \in [1,2]$ we establish an inversion formula arising from the Cesaro mean (i.e. the noncommutative generalization of the Fejer Theorem) and the Poisson average (i.e. the noncommutative generalization of the Abel Theorem).

Finally, in $L^2(M)$ we show how the corresponding one-parameter family of Fourier transforms "diagonalises" appropriately a one-parameter family of modular Dirac operators, which are part of a one-parameter family of genuine modular spectral triples.