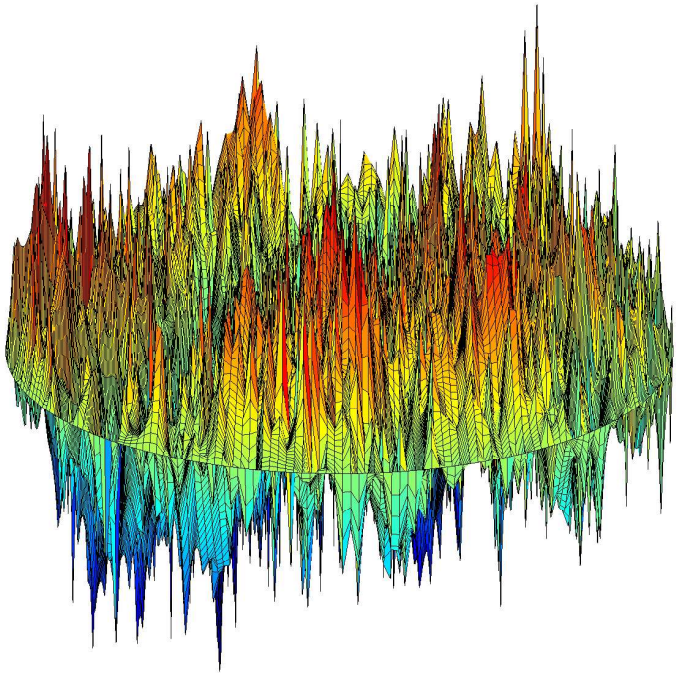
ICM 2014 Satellite Conference

Recent Progress in Random Conformal Geometry

August 11 – 12, 2014

Room 301B, COEX, Seoul, Korea



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The picture in the front page illustrates the graph of discrete Gaussian free field, borrowed from Nam-Gyu Kang and Nikolai G. Makarov, *Gaussian free field and conformal field theory*, Astrisque, **353** (2013), viii+136 pp.

Schedule

	August 11th (Monday)	August 12th (Tuesday)
08:30 AM	Registration	
09:00 AM	Gregory F. Lawler	Balint Virag
10:00 AM	Bertrand Duplantier	Vladas Sidoravicius
11:00 AM	Federico Camia	Pierre Nolin
11:30 AM		Kalle Kytölä
12:00 PM	Lunch	
02:00 PM	Alexander Vasiliev	Geoffrey Grimmett
03:00 PM	Fredrik Viklund	Hao Wu
03:30 PM	Michael Kozdron	Seung-Yeop Lee
04:00 PM	Coffee Break	
04:30 PM	Juhan Aru	Tom Alberts
05:00 PM	Demeter Kiss	Alexei Borodin
05:30 PM	Dinner	
06:00 PM		Conference Dinner

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Conformal invariance of the Green's function for loop-erased random walk

Gregory F. Lawler

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Abstract

If D is a simply connected complex domain containing the origin with two boundary points a, b and A_n is an approximating lattice domain, we consider the probability that the loop-erased random walk (LERW) in A_n from (points near to) a to b goes though the origin. We show that the normalized limit is given by the Schramm-Loewner evolution (SLE) Green's function prediction — indeed, there exists c_0, u such that the probability is

$$c_0\,(rn)^{-3/4}\,[\sin^3\theta + O(n^{-u})].$$

where r is the conformal radius of D with respect to the origin and θ is the argument of the origin in D with respect to a, b. I will discuss the relationship between this and questions about natural parametrization of LERW and SLE.
This is joint work with Christian Benes and Fredrik Viklund.

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Multifractality of Whole-Plane SLE

Bertrand Duplantier

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Abstract

We study the (average) integral means spectrum of unbounded whole-plane SLE and prove the existence of a phase transition at a certain moment order. The bulk SLE multifractal spectrum, as predicted by the author in 2000 and derived by Beliaev and Smirnov in 2005, crosses over there to a new spectrum rigorously established for a certain range of moments. It is related to radial SLE derivative exponents, and to non- standard SLE tip exponents obtained from quantum gravity. The integral means spectrum for complex moments is also predicted.
Joint work with Chi Nguyen, Nga Nguyen and Michel Zinsmeister, to appear in Annales Henri Poincaré.

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Brownian Loops, Cosmic Bubbles and Conformal Fields

Federico Camia

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Abstract

The Brownian loop soup, introduced by Lawler and Werner, is a conformally invariant Poissonian ensemble of planar loops with deep connections to the Schramm-Loewner Evolution (SLE) and Conformal Loop Ensembles (CLEs), and hence to the continuum scaling limit of various important models of statistical mechanics. In this talk, I will introduce and discuss a set of "operators" of the Brownian loop soup whose correlation functions behave like those of the "primary fields" of a conformal field theory (that is, they scale covariantly under conformal maps). This leads to the conjecture that those "operators" may define full-fledged conformal field theories related to the Brownian loop soup. I will also briefly explain how the study of such "operators" was inspired by similar objects introduced in an attempt to study a set of cosmological theories known as "eternal inflation," thus in a context very far from that of statistical mechanics. (Based on joint work with Alberto Gandolfi and Matthew Kleban. No prior knowledge of conformal field theory or cosmology will be assumed.)

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Slit holomorphic stochastic flows

Alexander Vasiliev

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Abstract

We study general slit Loewner chains in the unit disk, which in the stochastic case lead to slit holomorphic stochastic flows. Radial, chordal and dipolar SLE are classical examples of such flows. Our approach, however, allows to construct new processes of SLE type that possess conformal invariance and the domain Markov property. We propose a classification for the variety of such processes. As a byproduct, we also give sufficient conditions for a diffusion equation to generate flows of holomorphic endomorphisms of the disk, which is of its own interest. Joint work with Georgy Ivanov and Alexey Tochin.

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Lattice representations of the Virasoro algebra

Fredrik Viklund

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Abstract

The Virasoro algebra is an infinite dimensional Lie algebra that appears as a consequence of conformal invariance in Conformal Field Theory (CFT) descriptions of continuum limits of certain critical 2D lattice models. The structure of the Virasoro algebra is important and lies behind differential equations that can be solved to find correlation functions explicitly. In this sense, these CFTs are "exactly solvable" and can be used to predict many things about the discrete models. I will discuss recent work with C. Hongler (Columbia/EPFL) and K. Kytola (Helsinki) which uses discrete complex analysis to construct a concrete representation of the Virasoro algebra directly on the discrete level for the discrete Gaussian free field on a square grid. In other words, we show that the Virasoro algebra is actually present already on the discrete level and as a consequence we can, e.g., identify the central charge $c = 1$ without reference to a scaling limit.

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Some Partial Results on the Convergence of Loop-Erased Random Walk to SLE(2) in the Natural Parametrization

Michael Kozdron

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Abstract

We outline a strategy for showing convergence of loop-erased random walk on the two-dimensional square lattice to SLE(2), in the supremum norm topology that takes the time parametrization of the curves into account. The discrete curves are parametrized so that the walker moves at a constant speed determined by the lattice spacing, and the SLE(2) curve has the recently introduced natural time parametrization. Our strategy can be seen as an extension of the one used by G. Lawler, O. Schramm, and W. Werner to prove convergence modulo time parametrization. The crucial extra step is showing that the expected occupation measure of the discrete curve, properly renormalized by the chosen time parametrization, converges to the occupation density of the SLE(2) curve, the so-called SLE Green's function. Although we do not prove this convergence, we rigorously establish some partial results in this direction including a new loop-erased random walk estimate. Based on joint work with Tom Alberts and Robert Masson.

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SLE + GFF $\neq$ KPZ

Juhan Aru

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Abstract

In this talk we mix together the Gaussian free field (GFF), some Schramm-Loewner Evolutions (SLEs) and the Liouville measure. What comes out is maybe in the first sight a bit surprising - the usual KPZ relation does not hold for the SLE flow lines of the GFF. We will try to explain why this is true intuitively and determine a quantum fractal dimension of the flow lines. On the way we will obtain some rather precise results on the winding of the SLE curves.

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Large deviation bounds for the volume of the largest cluster in 2D critical percolation

Demeter Kiss

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Abstract

Let M_n denote the size of largest open cluster in critical site percolation on the hexagonal inside a box of size n. We sketch a proof that the probability of the event $M_n/E(M_n) > x$ has lower and upper bounds of the form $exp(-cx^\alpha)$ for some $c > 0$ and large n. Here $\alpha = 96/5$, thus improves a result of Borgs, Chayes, Kesten and Spencer who deduced an upper bound with $\alpha = 1$. Our technique is based on the recent work of Manolescu, Sidoravicius and the speaker, and apply for other two dimensional lattices and partially for higher dimensional percolation.

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Spectral measure of the random matrix operators

Bálint Virág

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Abstract

Random unitary matrices, and more generally, circular beta ensembles have a limit called the Brownian Carousel operator. It is a random Dirac operator, whose eigenvalues correspond to the bulk limits of random matrices. A version of this operator has spectral measure that is closely related Liouville gravity on the line. I will introduce the Brownian Carousel operator and explain the connection between the two random measures.

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Dynamic Phase Transition: Maximal Increasing Sequence with a Defect Line and the Slow Bond Problem

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Abstract

In joint work with R. Basu and A. Sly (Berkeley) we address the question of how a localized microscopic defect, especially if it is small with respect to certain dynamic parameters, affects the macroscopic behavior of a system? In particular we consider two classical exactly solvable models: Ulam's problem of the maximal increasing sequence and the totally asymmetric simple exclusion process. For the first model, using its representation as a Poissonian version of directed last passage percolation on $\mathbb{R}^2$, we introduce the defect by placing a positive density of extra points along the diagonal line. For the latter, the defect is produced by decreasing the jump rate of each particle when it crosses the origin.

The powerful algebraic tools for studying these models break down in the perturbed versions of the models. Taking a more geometric approach we show that in both cases the presence of an arbitrarily small defect affects the macroscopic behavior of the system: in Ulam's problem the time constant increases, and for the exclusion process the flux of particles decreases. This, in particular, settles the longstanding "Slow Bond Problem".

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Percolation on uniform infinite planar maps

Pierre Nolin

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Abstract

A lot of work has been devoted recently to studying large planar maps, with a view to understanding what is a "generic" planar geometry. In a pioneering work that we first discuss, Angel and Schramm constructed the Uniform Infinite Planar Triangulation (UIPT), obtained as the $n \to \infty$ limit of planar triangulations with n vertices, chosen uniformly at random.
In a similar way, we construct the Uniform Infinite Planar Map (UIPM), obtained from planar maps with n edges. We then describe how the UIPM can be sampled using a "peeling" process, similar to a process introduced by Angel for the UIPT. This process allows one to discover the UIPM edge by edge, and we use it to prove that for bond and site percolation on the UIPM, the percolation thresholds are $p_c^{\text{bond}} = 1/2$ and $p_c^{\text{site}} = 2/3$ respectively.
This talk is based on a joint work with Laurent Ménard (Universit Paris Ouest Nanterre).

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Hidden quantum group symmetry in conformal random geometry

Kalle Kytölä

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Abstract

Questions about random conformally invariant curves frequently require finding explicit conformally covariant correlation functions, which are determined by a system of partial differential equations and boundary conditions specifying asymptotic behaviors. In this talk, I present a systematic construction of such boundary correlation functions using representation theory of a quantum group, the q-deformation of the Lie algebra sl(2). As an application I analyze the question of multipoint boundary visit probabilities of chordal SLE. The talk is based on joint work with Eveliina Peltola [arXiv:1408.1384], with Niko Jokela and Matti Järvinen [arXiv:1311.2297], and work in progress with Konstantin Izyurov.

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Counting self-avoiding walks

Geoffrey Grimmett

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Abstract

The connective constant of a graph is the exponential growth rate of the number of self-avoiding walks. The theory of connective constants runs parallel to that of critical points of interacting systems. I shall summarise recent results (with Zhongyang Li) on equalities and inequalities for connective constants of vertex-transitive graphs.

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Intersections of SLE paths

Hao Wu

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Abstract

SLE curves are introduced by Oded Schramm as the candidate of the scaling limit of discrete models. In this talk, we first describe basic properties of SLE curves and their relation with discrete models. Then we summarize the Hausdorff dimension results related to SLE curves, in particular the new results about the dimension of cut points and double points. Third we introduce Imaginary Geometry, and from there give the idea of the proof of the dimension results.

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Topology of quadrature domains: sharp bound

Seung-Yeop Lee

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Abstract

We study the two-dimensional support of the equilibrium measure when the (derivative of) external potential is given by a rational function. Given a degree of the rational function, there exists a bound in the maximal number of components in the support. In this talk, we show that this bound is sharp, meaning that there exists a rational function that realizes the maximal bound. This is a joint work with Nikolai Makarov.

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Random Geometry in the Spectral Measure of the Circular Beta Ensemble

Tom Alberts

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Abstract

The spectral measure is a useful tool for studying unitary operators on a Hilbert space, and for random operators the corresponding random measure can have a very rich geometry. I will discuss the multifractal behavior that is observed in the spectral measure for the Circular Beta Ensembles, which is the collection of random matrices whose eigenvalue distribution is that of the Coulomb gas confined to the circle. This is joint work with Balint Virag and Raoul Normand.

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Gaussian Free Field in beta ensembles and random surfaces

Alexei Borodin

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Abstract

The goal of the talk is to argue that the two-dimensional Gaussian Free Field is a universal and unifying object for global fluctuations of spectra of random matrices and random surfaces. This viewpoint leads to natural Gaussian processes on larger spaces which, despite their explicit covariance structure, so far lack conceptual understanding.