Critical Phenomena and the Renormalisation Group

NIMS, June 2015

Differential form representation theorems for self-avoiding walk are reviewed in

- David C. Brydges, John Z. Imbrie, and Gordon Slade. Functional integral representations for self-avoiding walk. *Probab. Surv.*, 6:34–61, 2009

Results in four dimensions for continuous-time weakly self-avoiding walk and n-component lattice spin models dimensions are obtained in

saw-log Roland Bauerschmidt, David C. Brydges, and Gordon Slade. Logarithmic Correction for the Susceptibility of the 4-Dimensional Weakly Self-Avoiding Walk: A Renormalisation Group Analysis. *Comm. Math. Phys.*, 337(2):817–877, 2015

saw4 Roland Bauerschmidt, David C. Brydges, and Gordon Slade. Critical Two-Point Function of the 4-Dimensional Weakly Self-Avoiding Walk. *Comm. Math. Phys.*, 338(1):169–193, 2015

scaling Roland Bauerschmidt, David C. Brydges, and Gordon Slade. Scaling limits and critical behaviour of the 4-dimensional n-component $|\phi|^4$ spin model. *J. Stat. Phys.*, 157(4-5):692–742, 2014

The renormalisation group began in quantum field theory with

- M. Gell-Mann and F. E. Low. Quantum electrodynamics at small distances. *Physical Rev. (2)*, 95:1300–1312, 1954
- E. C. G. Stueckelberg and A. Petermann. La normalisation des constantes dans la theorie des quanta. *Helv. Phys. Acta*, 26:499–520, 1953

These papers proposed ordinary differential equations expressing how the coupling constants in quantum field theory change under scaling. In these lectures we will not assume you know or care about quantum field theory.

Ken Wilson reformulated the essential ideas in these papers within statistical mechanics. His Nobel prize lecture is published in

- Kenneth G. Wilson. The renormalization group and critical phenomena. *Rev. Mod. Phys.*, 55:583–600, Jul 1983

The first article to achieve rigorous control of errors in the Wilson RG is

- K. Gawedzki and A. Kupiainen. Block spin renormalization group for dipole gas and $(\bigtriangledown\phi)^4$. *Ann. Phys.*, 147:198, 1983

For existence and properties of finite range decompositions of gaussian random fields and earlier work see

- Roland Bauerschmidt. A simple method for finite range decomposition of quadratic forms and Gaussian fields. *Probab. Theory Related Fields*, 157(3-4):817–845, 2013

Hierarchical models appeared first under the name *Branching Random Walk* in genetics. They were first introduced into physics by

- F. Dyson. Existence of a phase transition in a one dimensional Ising ferromagnet. *Commun. Math. Phys.*, 12:91–107, 1969

We wrote an earlier review of the renormalisation group in

Park City David C. Brydges. Lectures on the renormalisation group. In *Statistical Mechanics*, volume 16 of *IAS/Park City Math. Ser.*, pages 7–93. Amer. Math. Soc., Providence, RI, 2009

Norms, localisation, perturbation theory and many other important topics are developed in detail in

RG1 David C. Brydges and Gordon Slade. A Renormalisation Group Method. I. Gaussian Integration and Normed Algebras. *J. Stat. Phys.*, 159(3):421–460, 2015

RG2 David C. Brydges and Gordon Slade. A Renormalisation Group Method. II. Approximation by Local Polynomials. *J. Stat. Phys.*, 159(3):461–491, 2015

RG3 Roland Bauerschmidt, David C. Brydges, and Gordon Slade. A Renormalisation Group Method. III. Perturbative Analysis. *J. Stat. Phys.*, 159(3):492–529, 2015

RG4 David C. Brydges and Gordon Slade. A Renormalisation Group Method. IV. Stability Analysis. *J. Stat. Phys.*, 159(3):530–588, 2015

RG5 David C. Brydges and Gordon Slade. A Renormalisation Group Method. V. A Single Renormalisation Group Step. *J. Stat. Phys.*, 159(3):589–667, 2015