Thur., August 7	11:50 $\sim$ 12:15	Mugunghwa	25min talk

Second order asymptotics for the block counting process in a Λ-coalescent

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Given a finite measure Λ on $[0,1]$, the associated Λ-coalescent $(\Pi_t, t \geq 0)$ is a process taking values in the set of partitions of $\{1, 2, \ldots\}$. Its law is specified by the requirement that, for any $n \in \mathbb{N}$, the restriction Π^n of Π to $\{1, \ldots, n\}$ is a continuous-time Markov chain with the following transitions: whenever Π^n has $b \in [2, n]$ blocks, any given k-tuple of blocks coalesces at rate $\lambda_{b,k} := \int_{(0,1]} r^{k-2}(1-r)^{b-k}\Lambda(dr)$. Processes of this type, also known as coalescents with multiple collisions, are related to genealogies in certain population models. They were introduced by Pitman (1999) and Sagitov (1999).

Let N_t denote the number of blocks at time t of a standard Λ-coalescent, starting from a partition of $\mathbb{N}$ into singletons. There are well known conditions under which the coalescent process "comes down from infinity", that is, N_t is a.s. finite for all $t > 0$. Moreover, under these conditions Berestycki et al. (2010) found the speed of coming down from infinity. This is a deterministic function v satisfying $N_t/v_t \to 1$ as $t \to 0$.

In the present talk we discuss the second-order asymptotics for $(N_t)_{t\geq 0}$ at small times. We show, that if the driving measure Λ has no atom at 0 and near 0 it has a density, which behaves as $x^{-\beta}$ with $\beta \in (0,1)$, then, as $\varepsilon \to 0$, the process

$$\varepsilon^{-1/(1+\beta)} \left(\frac{N_{\varepsilon t}}{v_{\varepsilon t}} - 1 \right)_{t \geq 0}$$

converges in law in the Skorokhod space to the process

$$Const \frac{1}{t} \int_0^t u dL_u, \qquad t \geq 0,$$

where L is a spectrally negative $(1+\beta)$-stable Lévy process. If Λ has an atom at 0 (which corresponds to a non-trivial Kingman part), then the appropriate scaling is $\varepsilon^{-\frac{1}{2}}$ and the limit process is Gaussian: $Const \frac{1}{t} \int_0^t u dW_u$, where W is a standard Brownian motion.

Joint work with Vlada Limic.

V. Limic, A. Talarczyk, "Second-order asymptotics for the block counting process in a class of regularly varying Λ-coalescents", arXiv:1304.5183 (to appear in Ann. Probab.)