Gerald Trutnau Department of Mathematical Sciences, SNU

Financial Mathematics 1 - Spring term 2015

Exercise sheet no.11 (28.5.2015)

Exercise 1: For a European call calculate the delta, gamma, theta, and the vega in the Black-Scholes model (cf. Remark 4.12 of the lecture).

Exercise 2: Consider the Black-Scholes model.

(i) Let $G \in C^{1,2}([0,T) \times \mathbb{R})$. Show that under P for any $t < T$

$$\begin{aligned} G(t, S_t) &= G(0, S_0) + \int_0^t \sigma S_s \partial_x G(s, S_s) dB_s \\ &\quad + \int_0^t \left(\frac{\sigma^2}{2} S_s^2 \partial_{xx} G(s, S_s) + \mu S_s \partial_x G(s, S_s) + \partial_t G(s, S_s) \right) ds. \end{aligned}$$

(ii) Let $F(t,x)$ be as in display (35) on page 65 of the lecture. Suppose we follow the (self-financing) replicating strategy determined by $H_t = \partial_x F(t, S_t)$ (Δ hedging) as defined on page 66/67 of the lecture. Show that (under P, not P^* !) for any $t < T$

$$d\widetilde{V}_t = \sigma \widetilde{S}_t \partial_x F(t, S_t) dB_t + (\mu - r) \partial_x F(t, S_t) \widetilde{S}_t dt.$$

(iii) Show that $\widetilde{V}_t = e^{-rt} F(t, S_t)$ also satisfies for any $t < T$

$$\begin{aligned} d\widetilde{V}_t = d(e^{-rt} F(t, S_t)) &= \sigma \widetilde{S}_t \partial_x F(t, S_t) dB_t \\ + \left(\frac{\sigma^2}{2} e^{-rt} S_t^2 \partial_{xx} F(t, S_t) + \mu \widetilde{S}_t \partial_x F(t, S_t) + e^{-rt} \partial_t F(t, S_t) - r e^{-rt} F(t, S_t) \right) dt. \end{aligned}$$

(iv) Show that for any $s < T$

$$rF(s, S_s) = \frac{\sigma^2}{2} S_s^2 \partial_{xx} F(s, S_s) + r \partial_x F(s, S_s) S_s + \partial_t F(s, S_s).$$

Please drop the solutions into the homework box of the lecture until 4.6.2015, 6 pm