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Financial Mathematics 1 - Spring term 2015

Exercise sheet no.7 (30.4.2015)

Exercise 1: Let $(\Omega, \mathcal{F}, P)$ be a probability space, and

$$\mathcal{N}^P := \{A \subset \Omega \,|\, A \subset B \in \mathcal{F},\ P(B) = 0\}.$$

The completion of $\mathcal{F}$ with respect to P is defined by

$$\mathcal{F}^P := \{C \cup A \,|\, C \in \mathcal{F},\ A \in \mathcal{N}^P\}.$$

Show that $\mathcal{F}^P$ defines a σ-algebra on Ω and that P extends to $\mathcal{F}^P$ by the formula $P(C \cup A) = P(C)$, if $C \in \mathcal{F}$, and $A \in \mathcal{N}^P$.

Exercise 2: Show Proposition 3.6 about stopping times of the lecture.

Exercise 3: (i) Let $X \sim N(m, \sigma^2)$, $m \geq 0$, $\sigma^2 > 0$. Show that $E[X] = m$, and $\mathrm{var}(X) = \sigma^2$.

(ii) Let $X \sim N(0, \sigma^2)$. Show that

$$E[X^{2n-1}] = 0, \text{ and } E[X^{2n}] = \sigma^{2n}(2n-1)(2n-3)\cdot \ldots \cdot 1 \quad \text{for all } n \geq 1.$$

Exercise 4: Let $((B_t)_{t\geq 0}, P)$ be a standard $(\mathcal{F}_t)_{t\geq 0}$-Brownian motion. Show that

$$((e^{\sigma B_t - \frac{\sigma^2}{2}t})_{t\geq 0}, (\mathcal{F}_t)_{t\geq 0}, P)$$

is a martingale for any $\sigma \in \mathbb{R}$.

Please drop the solutions into the homework box of the lecture until 7.5.2015, 6 pm