Gerald Trutnau Department of Mathematical Sciences, SNU

Financial Mathematics 2 - Fall term 2015

Exercise sheet no.4 (1.10.2015)

Exercise 1: Show that the sufficient condition for invertibility in Remark 5.16 of the lecture is satisfied in the Black-Scholes model, if $|r-\frac{\sigma^2}{2}| \leq \frac{\sigma^2}{h}$.

Exercise 2: As mentioned at the end of Section 5.3 of the lecture, derive the price formula of a European put in the Black-Scholes model using the put-call parity

$$C(t,x) - P(t,x) = x - Ke^{-r(T-t)}, \qquad \forall t < T, x > 0.$$

Exercise 3: (Cf. page 102 of the lecture) Verify that

$$\partial_t F = -K\frac{\sigma^2}{2}v_\tau \text{ and } \partial_{xx}F = \frac{e^{-2y}}{K}(v_{yy} - v_y).$$

Exercise 4: (Cf. page 104 of the lecture)

(i) Show that

$$\left(e^{\frac{k+1}{2}\left(y+\sqrt{2\tau}x\right)} - e^{\frac{k-1}{2}\left(y+\sqrt{2\tau}x\right)}\right)^+ > 0 \iff x > -\frac{y}{\sqrt{2\tau}}.$$

(ii) Show that

$$\frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}}\left(e^{\frac{k+1}{2}\left(y+\sqrt{2\tau}x\right)} - e^{\frac{k-1}{2}\left(y+\sqrt{2\tau}x\right)}\right)^+ e^{-\frac{x^2}{2}}dx$$
$$= \frac{e^{\frac{k+1}{2}y}e^{\frac{(k+1)^2\tau}{4}}}{\sqrt{2\pi}}\int_{-\frac{y}{\sqrt{2\tau}}}^{\infty} e^{-\frac{\left(x-\frac{k+1}{2}\sqrt{2\tau}\right)^2}{2}}dx - e^{\frac{k-1}{2}y}e^{\frac{(k-1)^2\tau}{4}}\int_{-\frac{y}{\sqrt{2\tau}}}^{\infty} e^{-\frac{\left(x-\frac{k-1}{2}\sqrt{2\tau}\right)^2}{2}}dx.$$

Please drop the solutions into the homework box of the lecture until 8.10.2015, 6 pm