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Financial Mathematics 2 - Fall term 2015

Exercise sheet no.5 (08.10.2015)

Exercise 1: Let $(M_t)_{t\in[0,T]}$ be a continuous martingale such that

$$P(M_t > 0) = 1, \quad \forall t \in [0,T].$$

Set $\tau := (\inf\{t \in [0,T] \mid M_t = 0\}) \wedge T$.

(i) Show that τ is a stopping time.

(ii) Using the optional sampling theorem, show that $E[M_T] = E[M_T 1_{\{\tau=T\}}]$. Deduce that $P(M_t > 0 \ \forall t \in [0,T]) = 1$.

Exercise 2: Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, P)$ be a filtered probability space and let Q be a probability measure absolutely continuous with respect to P. Denote by L_t the density of the restriction of Q to $\mathcal{F}_t$. Let $(M_t)_{t\in[0,T]}$ be an adapted process. Show that $(M_t)_{t\in[0,T]}$ is a martingale under Q, if and only if the process $(L_t M_t)_{t\in[0,T]}$ is a martingale under P.

Exercise 3: The notations are those of Section 6.0.2 of the lecture notes. Let $(M_t)_{t\in[0,T]}$ be a process adapted to the filtration $(\mathcal{F}_t)_{t\in[0,T]}$. Suppose that $(M_t)_{t\in[0,T]}$ is a martingale under P^*. Using Exercise 2, show that there exists an adapted process $(H_t)_{t\in[0,T]}$ such that $\int_0^T H_t^2 dt < \infty$ a.s. and

$$M_t = M_0 + \int_0^t H_s d\tilde{W}_s \quad \text{a.s.} \quad \forall t \in [0,T].$$

Please drop the solutions into the homework box of the lecture until 15.10.2015, 6 pm