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Financial Mathematics 2 - Fall term 2015

Exercise sheet no.6 (15.10.2015)

Exercise 1: Consider the situation of Remark 6.12 (page 118-119 of the lecture notes). In the following the notations are the same as in Remark 6.12.

(i) Show that

$$C_t^\theta = C_0^\theta + \int_0^t \frac{\partial B}{\partial x}(t, P^\theta(t,T))dP^\theta(t,T), \quad t \in [0,\theta].$$

Hint: Apply Ito's formula and recall that $(C_t^\theta)_{t\in[0,\theta]}$ is a P^θ-martingale.

(ii) Show that $\frac{\partial B}{\partial x}(t,x) = N(d_1(t,x))$.

(iii) Prove that $dC_t = H_t^T dP(t,T) + H_t^\theta dP(t,\theta)$. Explain why the option can be hedged by holding H_t^T (resp. H_t^θ) zero-coupon bonds with maturity T (resp. θ) at time t.

(iv) In the framework of the Vasicek model, prove that $\sigma_t^T = -\frac{\sigma(1-e^{-a(T-t)})}{a}$, where a is the mean reversion factor (cf. (93) of the lecture notes). Compute the value of the call option and the hedge ratios.

Exercise 2: Show (a) and (b) on page 122 of the lecture notes.

Exercise 3: Verify the last formulas given on page 123 of the lecture notes, i.e. show

$$P(t,T) = \exp\left[-(T-t)R(T-t, r(t))\right],$$

where

$$R(\theta, r) = R_\infty - \frac{1}{a\theta}\left[(R_\infty - r)(1-e^{-a\theta}) - \frac{\sigma^2}{4a^2}(1-e^{-a\theta})^2\right],$$

and

$$R_\infty = \lim_{\theta\to\infty} R(\theta, r) = b^* - \frac{\sigma^2}{2a^2}.$$

Explain why R_∞ can be interpreted as long-term rate.

Please drop the solutions into the homework box of the lecture until 22.10.2015, 6 pm