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Financial Mathematics 2 - Fall term 2015

Exercise sheet no.8 (12.11.2015)

Exercise 1: Show that

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B) \quad \forall A \in \mathcal{A}_0, B \in \mathcal{B}_0.$$

where $\mathcal{A}_0$, $\mathcal{B}_0$, are as on page 141 of the lecture notes implies (116) of the lecture notes. *Hint*: You may use Proposition 11.4 on page 62 of the lecture notes from TIM 1.

Exercise 2: Show that

$$\mathbb{P}(\{U_{N_s+j} \in \tilde{H}\} \cap \{d \leq N_s\}) = \mathbb{P}(U_j \in \tilde{H})\mathbb{P}(d \leq N_s) \text{ for any } j \geq 1, d \geq 1,$$

as seen on page 142 of the lecture notes implies that U_{N_s+j} and U_j have same law for any $j \geq 1$.

Exercise 3: Show that

$$\mathbb{E}\left[e^{\sigma(W_t - W_s) + (\mu - r - \frac{\sigma^2}{2})(t-s)} \left(\prod_{j=1}^{N_t - N_s} (1 + U_j)\right)\right] = e^{(\mu - r)(t-s)} \mathbb{E}\left[\prod_{j=N_s+1}^{N_t} (1 + U_j)\right]$$

as seen on page 143 of the lecture notes.

Exercise 4: Let $(V_j)_{j \geq 1}$ be a sequence of non-negative, independent and identically distributed random variables. Let N be a Poisson distributed random variable with parameter $\lambda > 0$, and independent of the sequence $(V_j)_{j \geq 1}$. Show that

$$\mathbb{E}[\Pi_{j=1}^{N} V_j] = \exp\left(\lambda(\mathbb{E}[V_1] - 1)\right).$$

Please drop the solutions into the homework box of the lecture until 19.11.2015, 6 pm