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Topics in Mathematics 1 - Spring term 2017

Exercise sheet no.10 (18.5.2017)

Exercise 1: Let μ be an absolutely continuous distribution with density f. Define the corresponding entropy through

$$H(\mu) = -\int f(x) \log f(x)\, dx.$$

Show the following:

(i) The exponential distribution with parameter m^{-1} has maximal entropy among all absolutely continuous probability distributions on $[0,\infty)$ with fixed expectation $m > 0$.

(ii) The standard normal distribution has maximal entropy among all absolutely continuous probability distributions on $\mathbb{R}$ with fixed variance 1.

Exercise 2: Show that the following statements are equivalent for a random variable X:

(i) The Laplace transform $Z(\lambda) = E[e^{\lambda X}]$ of X is finite in a neighborhood of 0.

(ii) There are $a, b > 0$ with $P(|X| \geq t) \leq ae^{-bt}$, $t \geq 0$.

Exercise 3: Show that for the random walk S_n from Exercise 2, Exercise sheet no.5:

$$\lim_{n\to\infty} P(S_n > an)^{\frac{1}{n}} = \frac{1}{\sqrt{(1+a)^{1+a}(1-a)^{1-a}}} \text{ for } 0 < a < 1 .$$

What does in case $a \geq 1$ hold ?

Please drop the solutions into the homework box for the lecture until 25.5.2017, 6 pm