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Topics in Mathematics 1 - Spring term 2017

Exercise sheet no.3 (23.3.2017)

Exercise 1: let P be the uniform distribution on the set Ω of all permutations of $\{1, ..., n\}$. For a permutation ω let $X(\omega)$ be the number of fixed points. Calculate the expectation and the variance of X.

Exercise 2: Let $X_1, X_2, ...$ be a sequence of random variables with $E[X_i] \equiv 0$ and $\mathrm{Var}(X_i) \equiv \sigma^2 < \infty$. Suppose $|\mathrm{cov}(X_i, X_j)| \leq r(|i-j|)$, $i \neq j$, for some function r on $\mathbb{N}$. Find a condition on r (i.e. a condition on the decay of the correlations) under which the weak law of large numbers

$$\lim_{n\to\infty} P\left(\left|\frac{X_1 + ... + X_n}{n} - m\right| \geq \varepsilon\right) = 0, \quad \forall \varepsilon > 0$$

still holds.

Exercise 3: (Bienaymé formula) Let $X_1, ..., X_n \in \mathcal{L}^2$ be pairwise uncorrelated. Show that then

$$\mathrm{Var}(\sum_{i=1}^n X_i) = \sum_{i=1}^n \mathrm{Var}(X_i).$$

Please drop the solutions into the homework box for the lecture at the basement of building no. 25 until 30.3.2017, 6 pm