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Topics in Mathematics 1 - Spring term 2017

Exercise sheet no.7 (27.4.2017)

Exercise 1: Let $X_1, X_2, ...$ be a sequence of independent and identically distributed random variables. Show that

$$\lim_{n\to\infty} \frac{X_n}{n} = 0 \text{ P-a.s.} \iff E[|X_1|] < \infty.$$

What can one state without assuming the independence of the $X_n, n \in \mathbb{N}$?
(*Hint: Show first that X_1 is integrable, iff for all $\varepsilon > 0$*

$$\sum_{n=1}^{\infty} P(|X_1| > n\varepsilon) < \infty.)$$

Exercise 2: Show for the random walk S_n from Assignment no.5 Exercise 3:

$$\limsup_{n\nearrow\infty} S_n = +\infty, \quad \liminf_{n\nearrow\infty} S_n = -\infty \quad P\text{-a.s.}$$

Exercise 3: Let X_1, X_2 be random variables on a probability space $(\Omega, \mathcal{A}, P)$ and $\overline{X} = (X_1, X_2)$. Consider the following cases:

(i) $\overline{X}$ is uniformly distributed on $[0,1]^2$.

(ii) $\overline{X}$ is uniformly distributed on the unit circle $\{(x_1, x_2) \mid x_1^2 + x_2^2 \leq 1\}$.

Are the random variables X_1, X_2 independent ? Determine also the distributions of X_1 and X_2.

Exercise 4: The density of the Gamma-distribution $\Gamma_{\alpha,p}$ ($\alpha > 0$, $p > 0$) is given by

$$\gamma_{\alpha,p}(x) = \frac{1}{\Gamma(p)} \alpha^p x^{p-1} e^{-\alpha x} \cdot 1_{(0,\infty)}(x)$$

With the help of II. Proposition 4.5 of the lecture determine the distribution of the sum of two independent random variables that are Γ_{α,p_1}- resp. Γ_{α,p_2}-distributed.

Please drop the solutions into the homework box for the lecture at the basement of building no. 25 until 4.5.2017, 6 pm