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Topics in Mathematics 1 - Spring term 2017

Exercise sheet no.8 (4.5.2017)

Exercise 1: Let X, Y be independent, $N(0, \sigma^2)$-distributed random variables and

$$R := \sqrt{X^2 + Y^2} \qquad \text{and} \qquad \Phi := \arctan \frac{Y}{X}.$$

Show the following:

(i) R and Φ are independent.

(ii) Φ is unifomly distributed on $(-\frac{\pi}{2}, \frac{\pi}{2})$.

(iii) The distribution of R is absolutely continuous with density $\frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \cdot 1_{[0,\infty)}(r)$.

Exercise 2: Let μ_n, μ be probability measures on $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$ with distribution functions F_n, F. Show: If (μ_n) converges weakly to μ and F is continuous, then (F_n) converges uniformly on $\mathbb{R}$ to F.

Exercise 3: Let (X_n) be a sequence of indepengent, square integrable random variables with variances $\sigma_n^2 > 0$ that has the CLP. Let $s_n := (\sum_{k=1}^n \sigma_k^2)^{1/2}$ and $\alpha_n := \varepsilon n / s_n$. Show with the help of Exercise 2, that

$$P\left(\left| \frac{1}{n} \sum_{i=1}^n (X_i - E[X_i]) \right| < \varepsilon \right) - \frac{1}{\sqrt{2\pi}} \int_{-\alpha_n}^{\alpha_n} e^{-x^2/2}\, dx$$

converges uniformly in $\varepsilon > 0$ to zero as $n \to \infty$. Conclude from this, that (X_n) does not satisfy the WLLN if the sequence (n/s_n) is bounded.

Please drop the solutions into the homework box for the lecture until 11.5.2017, 6 pm