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Topics in Mathematics 1 - Spring term 2017

Exercise sheet no.9 (11.5.2017)

Exercise 1: Show with the help of the CLT, that

$$\lim_{n\to\infty} e^{-n} \sum_{k=1}^{n} \frac{n^k}{k!} = \frac{1}{2} \quad \text{and} \quad \lim_{n\to\infty} \frac{1}{(n-1)!} \int_0^n t^{n-1} e^{-t}\, dt = \frac{1}{2}.$$

Exercise 2: Show that for the random walk (cf. Exercise sheet no.5, Exercise 3)

$$\limsup_{n\to\infty} \frac{S_n}{\sqrt{n}} = +\infty, \qquad \liminf_{n\to\infty} \frac{S_n}{\sqrt{n}} = -\infty \qquad P\text{-a.s.}$$

Exercise 3: Let $\lambda \in \mathbb{R}$ and (X_n) be a a sequence of independent real random variables such that

$$P[X_n = n^\lambda] = P[X_n = -n^\lambda] = \frac{1}{2} \qquad (n \in \mathbb{N}).$$

Show:

(i) For $\lambda < -1/2$ the sequence (X_n) does not satisfy the Feller condition (2.5) in Lemma 6.4 of the lecture.

(ii) For $\lambda \geq -1/2$ we have $s_n^2 \geq \int_1^{n+1} x^{2\lambda}\, dx$ if $\lambda \leq 0$, and $s_n^2 \geq \int_0^n x^{2\lambda}\, dx$ if $\lambda \geq 0$.

(iii) (X_n) has the CLP for all $\lambda \geq -1/2$.

(iv) Show with the help of Exercise 3 (Assignment no. 8), that the sequence (X_n) does not satisfy the WLLN if $\lambda \geq 1/2$.

Please drop the solutions into the homework box for the lecture until 18.5.2017, 6 pm