Introduction to Stochastic Differential Equations

Course Syllabus

Fall Term 2009 — SNU

Course Title	Introduction to Stochastic Differential Equations (in English)
Course number	3341.352
Instructor	Gerald Trutnau
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Course homepage	http://www.math.snu.ac.kr/∼ trutnau/teachingSDE.html
Course Objective	The purpose of this course is to introduce the reader to the basic ideas and results of Stochastic Calculus up to the point that he can acquire a sufficient knowledge for the understanding of its role in applications.
References	There will be no main textbook. Here are some references: - Chung, K.L., Williams R.J.: *Introduction to stochastic integration*, second ed., Probability and its Applications, Birkhäuser Boston Inc., MA, 1990. - Durrett, Richard: *Stochastic calculus. A practical introduction.* Probability and Stochastics Series. Boca Raton, FL: CRC Press. - Karatzas, Ioannis; Shreve, Steven E.: *Brownian motion and stochastic calculus.* 2nd ed. Graduate Texts in Mathematics, Springer-Verlag. - Revuz, Daniel; Yor, Marc: *Continuous martingales and Brownian Motion.* 3rd ed. Grundlehren der Mathematischen Wissenschaften 293. Berlin: Springer. - L. C. G. Rogers & D. Williams: *Diffusions, Markov Processes and Martingales Vol 1* (Foundations) and *Vol 2* (Itô Calculus), Cambridge University Press.

Description Stochastic Calculus is in general considered as a course in the 4th academic year and is usually taught after two preceding probability theory courses (1 year). For example the reader is usually assumed to be familiar with martingale theory and conditional distributions and expectations on the probabilistic side but also with advanced measure theory as well as partial differential equations on the analytic side. However, we will try to present an understandable introduction focusing on concepts as well. We shall provide proofs or outlines of proofs whenever it seems to be possible without too much advanced theoretical machinery or whenever necessary for the illumination of a concept. The reader is assumed to have some reasonable background in advanced calculus, measure theory, and basic probability theory, though nonetheless we will always repeat and introduce at least the corresponding statements.

Tentative content (not necessarily in chronological order + there will also be an updating) Review of Probability, Pathwise Itô-Calculus, Kolmogorov's construction and properties of Brownian motion, continuous time martingale theory and stochastic integration, Itô's representation theorem, d-dimensional Itô formula and covariation, applications of Itô's formula, Girsanov formula and Novikov's condition, representation of continuous martingales as time-changed Brownian motion, stochastic processes, stochastic differential equations, weak and strong solutions, existence and uniqueness of solutions, Markov processes and semigroups, infinitesimal generator, one dimensinal diffusions, Bessel and Cox-Ingersoll-Ross process, Feynman-Kac formula. As time permits we may also treat some applications.

Teaching Method Lecture, exercises.

Evaluation - Attendance (10 % of final score).

- Assignment sheets (30 % of final score);

Students must solve exercises regularily, and will be given assignment sheets mostly every week.

- Midterm (8th week, 75 minutes, 30 % of final score);

- Final exam (15th week, 75 minutes, 30 % of final score);