TOPOLOGY I

Exercise sheet no.10

07.07.2005

Exercise 1: Let $A \neq \emptyset$. Show that the (long) reduced homology sequence

$$\cdots \xrightarrow{j_*} H_q(X,A) \xrightarrow{\partial_*} H_{q-1}(A) \xrightarrow{i_*} H_{q-1}(X) \xrightarrow{j_*} H_{q-1}(X,A) \xrightarrow{\partial_*} \ldots$$

$$\cdots \xrightarrow{j_*} H_1(X,A) \xrightarrow{\partial_*} H_0^{\#}(A) \xrightarrow{i_*} H_0^{\#}(X) \xrightarrow{j_*} H_0(X,A) \xrightarrow{\partial_*} 0$$

(here $q \geq 2$) is exact.

Exercise 2: We have already seen that S^n, $n \geq 1$, is simply connected. Is it contractible?

Exercise 3: Show the Brouwer fixed-point theorem: Let $f : D^n \to D^n$, $n \geq 1$, be continuous. Then there is $x \in D^n$ with $f(x) = x$. (Hint: Show that S^n cannot be a retract of D^{n+1}, $n \geq 1$.)

Exercise 4: Consider continuous maps $f, g : S^n \to S^n$, $n \geq 1$, such that $f(x) \neq g(x)$ for all $x \in S^n$. Show that:
(i) f is homotopic to $a \circ g$, where a is the antipodal map. In particular $H_n(f) = (-1)^{n+1} H_n(g)$.
(ii) If f is nullhomotopic, i.e. homotopic to a constant map, then f has a fixed point.
(iii) If $f : S^{2n} \to S^{2n}$, $n \geq 1$, is continuous, then either f has a fixed point or some point is sent to its antipode.

Exercise 5: Let $f : S^n \to S^n$, $n \geq 1$, $deg(f) \neq 0$. Then f is surjective.