TOPOLOGY I

Exercise sheet no.3

02.05.2005

Exercise 1: Denote by $\mathrm{int}(C)$ the interior of C. Let $A, B \subset X$, A connected. Let $A \cap \mathrm{int}(B) \neq \emptyset$, and $A \cap (X \setminus B) \neq \emptyset$. Show that $A \cap \partial B \neq \emptyset$.

Exercise 2: Show that a connected space needs not necessarily be path-connected.

Exercise 3: Show that $\mathbb{R}$ is not homeomorphic to $\mathbb{R}^2$.

Exercise 4: Let X be a Hausdorff space. Let $x, y \in X$ be a limit of the sequence $(x_n)_{n \in \mathbb{N}}$ in X. Show that $x = y$.

Exercise 5: Let $f : X \to Y$ be a continuous map. Show that the graph $G = \{(x, y) \in X \times Y | y = f(x)\}$ of f is homeomorphic to X.

Exercise 6: Show that a topological space X is T_4 if and only if for every closed subset A of X and every neighborhood U_A of A there is a neighborhood V_A of A with $\overline{V}_A \subset U_A$.