TOPOLOGY I

Exercise sheet no.7

10.06.2005

Exercise 1: Suppose (although this is not possible as we have seen) that $f : D^2 \to D^2$ is continuous such that $f(x) \neq x$ for every $x \in D^2$. Then for each $x \in D^2$ the line passing through x and $f(x)$ intersects S^1 at a unique point on the side of x. Denote this point by $r(x)$. Show that $r : D^2 \to S^1$ is continuous.

Exercise 2: Show that S^n, $n \geq 1$ is path-connected.

Exercise 3: Show that $\mathbb{R}^n$ is not homeomorphic to $\mathbb{R}^2$ for $n > 2$.

Exercise 4: Calculate:
(a) $\pi_1(S^1 \times I)$
(b) $\pi_1(\mathbb{R}^3 \setminus \{(0,0,z) : z > 0\})$
(c) $\pi_1(\mathbb{R}^3 \setminus \{(0,0,z) : z \in \mathbb{R}\})$

Exercise 5: Explain why $\pi_1(S^1) \cong \pi_1(\mathcal{M})$, where $\mathcal{M}$ is the Moebius strip.

Exercise 6: Let $C = \{(x,y) \in I \times I : x = 0, \text{ or } y = 0, \text{ or } y = \frac{1}{n}, n \in \mathbb{N}^*\}$. Show that C is contractible.