TOPOLOGY I

Exercise sheet no.8

23.06.2005

Exercise 1: Let $A = \{a_{ij}\}_{1 \le i,j \le 3}$ be a 3×3-matrix of strictly positive real numbers, i.e. $a_{ij} > 0$ for each $1 \le i, j \le 3$. Then A has a positive real eigenvalue (characteristic value).
Hint: Show first:
(a) If B is homeomorphic to D^2 and $f : B \to B$ is continuous, then f has a fixed-point.
(b) Consider the map $x \mapsto \frac{A(x)}{|A(x)|}$, on $B := S^2 \cap \{(x_1, x_2, x_3) | x_1 \ge 0, x_2 \ge 0, x_3 \ge 0\}$. Has it a fixed-point?

Exercise 2: Calculate explicitely $\partial_1 \partial_2(\delta_2)$ without using Lemma 1.4. of the lecture.

Exercise 3: (*Reduced homology*) Using the homomorphism

$$\partial^{\#} : S_0(X) \to \mathbb{Z}; \quad \partial^{\#}(\Sigma_x \nu_x x) = \Sigma_x \nu_x$$

one defines the 0-th reduced homology group $H_0^{\#}(X) := ker(\partial^{\#})/im(\partial_1)$.
Show that:
(a) $\partial^{\#} \circ \partial_1 = 0$
(b) $H_0^{\#}(X)$ is a free abelian group of $r - 1$ generators, where r is the number of path-connected components of X, so that the term "reduced" is justified.
For $q > 0$ we define $H_q^{\#}(X) = H_q(X)$.

Exercise 4: Show that the homology groups are topological invariants, i.e. if $f : X \to X'$ is a homeomorphism, then $H_q(f) : H_q(X) \to H_q(X')$ is an isomorphism.