TOPOLOGY I

Exercise sheet no.9

01.07.2005

Exercise 1: Let σ be a singular q-simplex in X. Show that

$$(id \times \sigma^{(j)}) \circ (A_0, ..., A_i, B_i, ..., B_{q-1})$$

$$= \begin{cases} (id \times \sigma) \circ (A_0, ..., A_i, B_i, ..., B_q) \circ F_{q+1}^{j+1} & \text{for } \quad 0 \leq i < j \leq q \\ (id \times \sigma) \circ (A_0, ..., A_{i+1}, B_{i+1}, ..., B_q) \circ F_{q+1}^{j} & \text{for } 0 \leq j \leq i \leq q-1. \end{cases}$$

(The notations are the same as in the lecture)

Exercise 2: Let $f : X \to Y$ be a homotopy equivalence. Show that $H_q(X) \cong H_q(Y)$ for every $q \in \mathbb{Z}$.

Exercise 3: Let $f : (X, A) \to (X', A')$. Show that

$$S_q(f)\,(Z_q(X, A)) \subset Z_q(X', A') \quad \text{and} \quad S_q(f)\,(B_q(X, A)) \subset B_q(X', A')$$

Exercise 4: Let $f, g : (X, A) \to (X', A')$ be homotopic, i.e. $f, g : X \to X'$ are homotopic and $f(A), g(A) \subset A'$. Show that then $H_q(f) = H_q(g) : H_q(X, A) \to H_q(X', A')$.

Exercise 5: Let $A \neq \emptyset$, $A \subset X$, X path-connected. Show that $H_0(X, A) = 0$.

Exercise 6: Let $A = \{x_0\}$, $x_0 \in X$. Show that $H_0(X, x_0) = \mathbb{Z}^{r-1}$ if X has r path-connected components.