Abstracts of KOTAC
Volume 10(2008), 11–11

Exact associate of C^*-tensor norm

Kyung Hoon Han
Department of Mathematics, Seoul National University, Korea
kyunghoon.han@gmail.com

The maximal tensor norm satisfies the following three conditions with $\alpha = \max$.

1. For C^*-algebras A_1, A_2, B_1, B_2 and completely positive maps $T_1 : A_1 \to B_1$ and $T_2 : A_2 \to B_2$, their tensor product $T_1 \otimes T_2$ is continuous with respect to C^*-tensor norm α with
$$\|T_1 \otimes T_2 : A_1 \otimes_\alpha A_2 \to B_1 \otimes_\alpha B_2\| \leqq \|T_1\| \|T_2\|,$$
and its continuous extension $T_1 \otimes T_2$ is completely positive.

2. For C^*-algebras A, B and an element z in their algebraic tensor product $A \odot B$, we have
$$\|z\|_{A \otimes_\alpha B} = \inf\{\|z\|_{C \otimes_\alpha D} : z \in C \odot D\}$$
where the infimum runs over a separable C^*-subalgebra C of A and a separable C^*-subalgebra D of B.

3. For C^*-algebras A, B and a norm closed ideal I of A, two sequences
$$0 \to I \otimes_\alpha B \to A \otimes_\alpha B \to A/I \otimes_\alpha B \to 0$$
and
$$0 \to B \otimes_\alpha I \to B \otimes_\alpha A \to B \otimes_\alpha A/I \to 0$$
are exact.

The C^*-tensor norm satisfying condition (1) need not satisfy condition (3). The tensor product of completely positive maps between C^*-algebras is continuous with respect to the minimal tensor norm, and its continuous extension is completely positive. Wasserman has proved that the sequence

$$0 \to I \otimes_{\min} C^*(\mathbb{F}_2) \to C^*(\mathbb{F}_2) \otimes_{\min} C^*(\mathbb{F}_2) \to C^*_r(\mathbb{F}_2) \otimes_{\min} C^*(\mathbb{F}_2) \to 0$$

is not exact for the free group $\mathbb{F}_2$. Hence, the minimal tensor norm does not satisfy condition (3) with $\alpha = \min$.

The maximal tensor norm is the largest C^*-tensor norm. For a C^*-tensor norm α satisfying condition (1), there exists at least one C^*-tensor norm larger than α that satisfies conditions (1),(2), and (3), for example the maximal tensor norm. It is natural to search for the smallest C^*-tensor norm among them and to determine its uniqueness. We will answer this question in the affirmative. It is a C^*-algebraic analogue of a projective associate in Banach space theory.