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A bluffer's guide to Weyl's theorem

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In 1909, motivated by questions of stability for a differential operator under changes of boundary conditions, Hermann Weyl showed that a compact perturbation of a self-adjoint operator leaves the spectrum invariant, except for isolated eigenvalues of finite multiplicity. In 1935 Von Neumann showed that this was the only possible change: if two self-adjoint operators have the same spectrum, up to such eigenvalues, then they are unitarily equivalent up to a compact perturbation. Weyl's result easily extended to normal operators, but von Neumann's converse resisted until 1970, when Berg and Sikonia independently supplied the missing ingredient: the extension to the normal case of Weyl's theorem that any self-adjoint operator may be compactly perturbed so as to have a complete set of eigenvectors, with no isolated eigenvalues of finite multiplicity.

In the late sixties and early seventies a number of authors, includeing Gustavson, Coburn, Berberian and later Oberai, focussed attention on a version of Weyl's theorem which said, for certain kinds of operator, that the spectrum was the disjoint union of these isolated eigenvalues of finite multiplicity and the "Weyl spectrum", intersection of the spectrums of all the compact perturbations of the original operator.

So "Weyl's theorem", which in the first instance, "holds for" self adjoint and then normal operators on Hilbert space, has been extended to more and more classes of bounded operators, on Banach as well as Hilbert spaces, and in the process has been subdivided into different components. It is our aim here to try and classify these, and reach down to what is really going on.