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Extreme points of the closed convex hull of composition operators on H^∞

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Let $S(D)$ be the set of all analytic self-maps of the open unit disk D and H^∞ be the set of all bounded analytic functions on D. Every analytic self-map $\varphi \in S(D)$ induces the composition operator C_φ on H^∞ defined by

$$C_\varphi f(z) = f(\varphi(z)),$$

which acts boundedly on H^∞. More precisely, we can see that $\|C_\varphi\|_{H^\infty} = 1$ for any $\varphi \in S(D)$.

We recall that an element x of a convex set X is called an extreme point of X if the conditions $0 < r < 1$, $x_1, x_2 \in X$ and $x = (1-r)x_1 + rx_2$, implies that $x_1 = x_2 = x$. By de Leeuw-Rudin's Theorem, it was proved that φ is an extreme point of the closed unit ball U_{H^∞} of H^∞ if and only if

$$\int_0^{2\pi} \log(1 - |\varphi(e^{i\theta})|)d\theta = -\infty.$$

Let $\mathcal{C}(H^\infty)$ be the collection of all bounded composition operators on H^∞, endowed with the operator norm topology. Then $\mathcal{C}(H^\infty)$ is a semigroup with respect to the products, but is not a linear space. Through the study on the topological structure of $\mathcal{C}(H^\infty)$, it was proved that C_φ is isolated in $\mathcal{C}(H^\infty)$ if and only if φ is an extreme point of the closed unit ball U_{H^∞} of H^∞.

On the other hand, from the fact that $\mathcal{C}(H^\infty)$ is not a linear space we derive the following problem:

Problem. When the linear combination of composition operators is a composition operator?

It is easy to show that there is no finite linear combination of composition operators which is a composition operator.

In this talk, we consider this problem for the case of the infinite linear combinations. We also consider the relationship between the extremeness of C_φ in the closed convex hull of $\mathcal{C}(H^\infty)$ and the extremeness of φ of U_{H^∞}.