

Multiplicativity of Cullis' determinant for rectangular matrices

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A definition of the determinant $\det_{n,k}(X)$ of an $n \times k$ rectangular matrix X with $n \geq k$ was first introduced by C. E. Cullis in the book published in 1913 from Cambridge University Press. Since this determinant is multilinear for column vectors of the rectangular matrix but not for row vectors, many of important formulae known for square matrices do not necessarily hold for these rectangular matrices. However we can show several expansion formulae for Cullis' determinant, such as an expansion for a column vector, a Laplace expansion and so forth. One of important properties of square matrices is deduced from the multiplicativity. In case of rectangular matrices, the right multiplicativity $\det_{n,k}(XA) = \det_{n,k}(X) \det_{k,k}(A)$ does hold, but does not the left multiplicativity $\det_{n,k}(BX) = \det_{n,n}(B) \det_{n,k}(X)$. In many applications, invariance of the determinant of a square matrix under the adjoint actions $X \mapsto BXB^{-1}$ plays important roles, since it induces many coordinate free invariants. In this talk we would like to consider the same type of problems for Cullis' determinant in connection with those for square matrices.

References

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