Abstracts of KOTAC
Volume 10(2008), 25–25

Stable rank for inclusions of C*-algebras

Hiroyuki Osaka
Department of Mathematical Sciences, Ritsumeikan University, Kusatsu, Shiga, 525-8577 Japan
osaka@se.ritsumei.ac.jp

Let A be a unital C*-algebra. We recall that the *topological stable rank* $\mathrm{tsr}(A)$ of A is defined to be the least integer n such that the set $\mathrm{Lg}_n(A)$ of all n-tuples $(a_1, , a_2 \dots, a_n) \in A^n$ which generate A as a left ideal is dense in A^n. The topological stable rank of a nonunital C*-algebra is defined to be that of its smallest unitization. Note that $\mathrm{tsr}(A) = 1$ is equivalent to density of the set of invertible elements in A. Furthermore, $\mathrm{tsr}(A) = 1$ implies that $\mathrm{tsr}(A \otimes M_n(\mathbb{C})) = 1$ for all n, and that $\mathrm{tsr}(A \otimes K) = 1$, where K is the algebra of compact operators on a separable infinite dimensional Hilbert space.

Blackadar proposed the question whether $\mathrm{tsr}(A \times_\alpha G) = 1$ for any unital AF C*-algebra A, a finite group G, and an action α of G on A. We prove that if a unital C*-algebra A has a simple unital C*-subalgebra D of A with common unit such that D has Property (SP) and $\sup_{p\in P(D)} \mathrm{tsr}(pAp) < \infty$, then $\mathrm{tsr}(A) \leq 2$. As an application let A be a simple unital C*-algebra with $\mathrm{tsr}(A) = 1$ and Property (SP), $\{G_k\}_{k=1}^n$ finite groups, α_k actions from G_k to $\mathrm{Aut}((\cdots((A \times_{\alpha_1} G_1) \times_{\alpha_2} G_2) \cdots) \times_{\alpha_{k-1}} G_{k-1})$. ($G_0 = \{1\}$) Then

$$\mathrm{tsr}((\cdots((A \times_{\alpha_1} G_1) \times_{\alpha_2} G_2) \cdots) \times_{\alpha_n} G_n) \leq 2.$$

We also present several examples which are affirmative data to the Blackadar's problem.