Abstracts of KOTAC
Volume 10(2008), 27–27

The real rank for C^*-algebra composition series

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The real rank for C^*-algebras is introduced by Brown and Pedersen as a real analogue of the (topological) stable rank for C^*-algebras introduced by Rieffel (that can be considered as the same as the (algebraic) Bass stable rank for rings). The real and stable ranks can be considered as real and complex dimensions for C^*-algebras from the basic formulae on commutative C^*-algebras, so that their theory is viewed as a noncommutitive dimension theory. Some basic formulae on the stable rank are obtained by Rieffel, and those on the real rank by Brown-Pedersen, especially in the case of real-rank-zero C^*-algebras. Since then, there are many contributions concerning with these basic and important concepts in C^*-algebra theory. Roughly speaking, the stable rank theory is rather well developed than the real rank theory. As for this, for instance, no formula on (general) C^*-algebra extensions as in the stable rank case had been a weak point in the real rank case. This situation is improved by the formula obtained by Nagisa, Osaka, and Phillips, that is applied to pull-back C^*-algebras associated with C^*-algebra extensions. But, their formula is not fully enough to estimate the real rank for C^*-algebra composition series since the multiplier algebras of their subquotients must be involved in their associated pull-back C^*-algebras, and computing their real rank should become another task. However, restricting our attension to CCR (or liminary) C^*-algebras and further imposing some assumptions on their spectrums, we are able to estimate their real rank and obtain its applications to the group C^*-algebras of CCR locally compact groups such as connected nilpotent Lie groups and semi-simple Lie groups. This is the first thing that I would like to announce here and its paper will appear in Kyungpook Math. J. The second (main) thing is a quite recently discovered new formula on the real rank estimate for C^*-algebra extensions as mentioned above and as desired. Now we are able to apply it to estimating the real rank for C^*-algebra composition series (in general), as well as type I C^*-algebras, that have composition series of closed ideals such that their subquotients have continuous trace. Also the main formula is obtained in studying the structure for certain semigroup crossed products of Toeplitz algebras and computing their real and stable ranks. However, there is a weak (but reasonable) point in it since it involves the connected stable rank of Rieffel, which must be also taken into consideration on the stable rank estimate for C^*-algebra extensions.