.

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ABSTRACTS

Operator Theory and Its Applications

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TABLE OF CONTENTS

Abstracts of KOTAC
Volume 15(2016), 1–1

Wavelets associated to representations of higher-rank graph algebras

Judith A. Packer
University of Colorado, U.S.A.
packer@colorado.edu

Here we discuss notions of wavelets defined on L^2-spaces for fractal-like sets associated to certain representations of higher-rank graph C^*-algebras, where the graphs in question are finite and strongly connected. We generalize work of M. Marcolli and A. Paolucci for Cuntz-Krieger C^*-algebras and obtain the wavelets and frames using the isometries and partial isometries that generate the C^*-algebras in question. This work is joint with C. Farsi, E. Gillaspy, and S. Kang.

Abstracts of KOTAC
Volume 15(2016), 2–2

Free probability on C*-algebras induced by Hecke algebras

Il Woo Cho
St. Ambrose University, U.S.A.
choilwoo@sau.edu

Hecke algebras are one of the main objects in modern number theory. They are used for studying automorphic forms, and (global or local) Adelic theory in analytic number theory. Here, we establish suitable free probabilistic structures on Hecke algebras, and study relations between number-theoretic information and our free-distributional data. Also, we construct suitable free-probabilistic Hilbert-space representations on Hecke algebras, and study corresponding C*-algebras generated by the Hecke algebras on the realized Hilbert spaces.

Abstracts of KOTAC
Volume 15(2016), 3–3

Sextic moment problems with a reducible cubic column relation

Seonguk Yoo
Sungkyunkwan University, Korea
seyoo73@gmail.com

Sextic moment problems with a single cubic column relation still needs to be investigated. Some specific cases are solved by L. Fialkow. In previous work the speaker studied the problem with a single cubic column relation corresponding to 3 parallel lines; we would like to extend the argument to a much wider class, the problem with reducible cubic column relations. It is anticipated that this result may contribute to a further analysis of nonsingular sextic moment problems through a rank-one deconposition of the associated moment matrix.

Abstracts of KOTAC
Volume 15(2016), 4–4

Common reducing subspaces of several weighted shifts with operator weights

Caixing Gu
Califonia Polytechnic State University, U.S.A.
cgu@calpoly.edu

We characterize common reducing subspaces of several weighted shifts with operator weights. As applications, we study the common reducing subspaces of the multiplication operators by powers of coordinate functions on Hilbert spaces of holomorphic functions in several variables. The identification of reducing subspaces also leads to structure theorems for the commutants of von Neumann algebras generated by these multiplication operators. This general approach applies to weighted Hardy spaces, weighted Bergman spaces, Drury-Arveson spaces and Dirichlet spaces of the unit ball or polydisk uniformly.

Abstracts of KOTAC
Volume 15(2016), 5–5

Recent results in Fredholm theory

Dragan Djordjevic
University of Nis, Serbia
dragandjordjevic70@gmail.com

We present new results in Fredholm theory, and some old results with different proofs.

Abstracts of KOTAC
Volume 15(2016), 6–7

On complex symmetric block Toeplitz operators

Eungil Ko and Ji Eun Lee*
Ewha Womans University, Sejong University, Korea
eiko@ewha.ac.kr, jieunlee7@sejong.ac.kr

In this paper, we study conjugation matrices and complex symmetric block Toeplitz operators. In particular, we examine conditions for 2×2 operator matrices to be conjugations or complex symmetric on $\mathcal{H} \oplus \mathcal{H}$. Using these results, we provide a characterization of complex symmetric block Toeplitz operators with some conjugations.

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Abstracts of KOTAC
Volume 15(2016), 8–8

A Wolff-type theorem in the Dirichlet space setting

Hyun-Kyoung Kwon
University of Alabama, U.S.A.
hkwon@ua.edu

A recent result of D. P. Banjade and T. T. Trent states that for $F = (f_1, f_2, \cdots)$, where $f_j \in \mathcal{M}(\mathcal{D})$, the multiplier algebra of the Dirichlet space, and

$$|H(z)| \leq (\sum_j |f_j(z)|^2)^{\frac{1}{2}},$$

for all $z \in \mathbb{D}$, there exists a $G = (g_1, g_2, \cdots)$ with $g_j \in \mathcal{M}(\mathcal{D})$ such that

$$FG^T = H^3.$$

With an additional relationship between F and H, we show how one can recover the function H. This talk is based on joint work with D. P. Banjade and T. A. Neal.

Abstracts of KOTAC
Volume 15(2016), 9–9

Generalized inverses and spectral sets

Gabriel Kantun-Montiel
Benemerita Universidad Autonoma de Puebla, Mexico
gkantun@fcfm.buap.mx

Generalized invertibility is naturally linked to spectral sets. The group, Drazin and Koliha-Drazin inverses are related to the spectral set $\{0\}$. In case of an (isolated) spectral set σ, we have the σ-g-Drazin inverse of Koliha and Dajic. These inverses can also be studied in the framework of the inverse along an element or, more generally, the (b, c)-inverse. In this talk we pay special attention to generalized inverses related to circularly isolated spectral sets. By a work of Koliha and Poon, these spectral sets are related to a generalization of Mbekhta decomposition.

(This is a joint work with Slavisa Djordjevic.)

Abstracts of KOTAC
Volume 15(2016), 10–10

Reducing subspaces on the Dirichlet space

Shuaibing Luo
Hunan University, P.R. China
shuailuo2@126.com

Suppose T is a bounded operator on a Hilbert space $\mathcal{H}$, if a closed subspace $\mathcal{M}$ of $\mathcal{H}$ is invariant under both T and T^*, then $\mathcal{M}$ is called a reducing subspace of T on $\mathcal{H}$. Let B be a finite Blaschke product which is not a Möbius transform, and let T be M_B, the multiplication operator by B. When $\mathcal{H}$ is the Hardy space H^2, it is known that there is a one-to-one correspondence between the reducing subspaces of M_B on H^2 and the closed subspaces of $H^2 \ominus BH^2$ ([2]). When $\mathcal{H}$ is the Bergman space L_a^2, in 2012, Douglas, Putinar and Wang [1] showed that the number of minimal reducing subspaces of M_B on L_a^2 equals the number of connected components of the Riemann surface $B^{-1} \circ B$. When $\mathcal{H}$ is the Dirichlet space D, little is known about the reducing subspaces of M_B on D. When the order of B is greater that 2, it remains open when M_B is reducible on D. In this talk, we will discuss the structure of the reducing subspaces of M_B on D when the order of B is greater than 2.

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Abstracts of KOTAC
Volume 15(2016), 11–11

Operators A for which there exists a compact operator K such that $A+K$ is hereditarily polaroid

B.P. Duggal
University of Nis, Serbia
bpduggal@yahoo.co.uk

A Banach space operator $A \in B(\mathcal{X})$ is polaroid, $A \in (\mathcal{P})$, if the isolated points of the spectrum $\sigma(A)$ of A are poles of the (resolvent of the) operator; A is hereditarily polaroid, $A \in (\mathcal{HP})$, if every part of A (i.e., its restriction to a closed invariant subspace) is polaroid. Polaroid operators, and their perturbation by commuting compact perturbations, have been studied by a number of authors in the recent past. For example, if $N \in B(\mathcal{X})$ is a nilpotent operator which commutes with $A \in B(\mathcal{X})$, then A is polaroid if and only if $A+N$ is polaroid. (This, however, does not extend to non-nilpotent quasinilpotent commuting operators.) The perturbation of a polaroid operator by a compact operator may or may not effect the polaroid property of the operator: It is known that given a Hilbert space operator $A \in B(\mathcal{H})$, there exist compact operators K_0 and $K \in B(\mathcal{H})$ such that (i) $A+K_0$ is polaroid and (ii) $A+K$ is not polaroid. A natural extension of this problem is the question of whether there exist compact operators K_0 and $K \in (\mathcal{H})$ such that (i)' $A+K_0$ is hereditarily polaroid and (ii)' $A+K$ is not hereditarily polaroid. Here the answer to (ii)' is a "yes" [1, Theorem 5.2], but there is caveat to the answer to (i)' - the answer is "yes if the set $\Phi^+_{sf}(A) = \{\lambda \in \sigma(A) : A-\lambda$ is semi- Fredholm and $\mathrm{ind}(A-\lambda) > 0\} = \emptyset$". We prove that if $A, K \in B(\mathcal{X})$ with K compact, then $A+K$ hereditarily polaroid implies $\Phi^+_{sf}(A) = \emptyset$. Indeed, we prove that if $A \in B(\mathcal{H})$, then there exists a compact $K \in B(\mathcal{H})$ such that $A+K$ is hereditarily polaroid if and only if A has SVEP, *the single-value extension property*, on $\Phi_{sf}(A)$. A sufficient condition for operators $A \in B(\mathcal{X})$ to have SVEP on $\Phi_{sf}(A)$ is that the component $\Omega_a(A) = \{\lambda \in \Phi_{sf}(A) : \mathrm{ind}(A-\lambda) \leq 0\}$ is connected. It is seen that, given an operator $A \in B(\mathcal{H})$, a necessary and sufficient condition for there to exist a compact operator $K \in B(\mathcal{H})$ such that $A+K \in (\mathcal{HP})$ is that $\Omega_a(A)$ is connected.

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Abstracts of KOTAC
Volume 15(2016), 12–12

Equivalence of operator norm and essential norm on operator algebras

Il Bong Jung
Kyungpook National University, Korea
ibjung@knu.ac.kr

Let $\mathcal{H}$ be a separable, infinite dimensional, complex Hilbert space and let $\mathcal{L}(\mathcal{H})$ be the algebra of all bounded, linear operators on $\mathcal{H}$. We write $\mathbf{K}$ for the ideal of compact operators in $\mathcal{L}(\mathcal{H})$ and denote the quotient map $\mathcal{L}(\mathcal{H}) \to \mathcal{L}(\mathcal{H})/\mathbf{K}$ by π. For each $T \in \mathcal{L}(\mathcal{H})$ we write $\|T\|_e := \|\pi(T)\|$ (essential norm of T). And $\mathbb{A}$ denotes a unital, norm-closed, subalgebra of $\mathcal{L}(\mathcal{H})$ and $\mathbb{A}^{-W}$ the closure of $\mathbb{A}$ in the weak operator topology. If there is a transitive subalgebra $\mathbb{A}$ of $\mathcal{L}(\mathcal{H})$ such that $\mathbb{A}^{-W} \neq \mathcal{L}(\mathcal{H})$, is it true that $\|\ \|$ and $\|\ \|_e$ are equivalent norms on $\mathbb{A}$? This problem was suggested by C. Pearcy in 2005. Note that if the problem can be solved affirmatively, then every $S + K \in (\mathbf{S} + \mathbf{K})$ has a nontrivial invariant subspace, where $(\mathbf{S} + \mathbf{K}) = \{S + K : S$ is subnormal in $\mathcal{L}(\mathcal{H})$ and $K \in \mathbf{K}\}$. Thus it is strongly motivated to solve this problem affirmatively. In this talk we discuss some conditions for the equivalence of two norms $\|\ \|_e$ and $\|\ \|$ on an algebra $\mathbb{A}$. For examples, (i) each set $\Gamma_\alpha(y)$ is bounded if and only if $\|\ \|_e$ and $\|\ \|$ are equivalent on $\mathbb{A}$, where $\Gamma_\alpha(y) = \{Ay : A \in \mathbb{A},\ \|A\|_e \leq \alpha\}$ for $\alpha > 0$ and $y \in \mathcal{H}$, (ii) if there exists $y \in \mathcal{H}$ such that $\Gamma_\alpha(y)^-$ is bounded and has nonempty interior, then $\|\ \|_e$ and $\|\ \|$ are equivalent norms on $\mathbb{A}$. (This is a joint work with C. Foias, E. Ko and C. Pearcy.)

Abstracts of KOTAC
Volume 15(2016), 13–13

Generalized reverse Cauchy inequality and applications to operator means

Hiroyuki Osaka* and Yukihiro Tsurumi
Ritsumeikan University, Japan
osaka@se.ritsumei.ac.jp, ra0006hk@ed.ritsumei.ac.jp

It is well-known as Young inequality that for $0 \leq \nu \leq 1$ and $a, b \geq 0$

$$\nu a + (1-\nu)b \geq a^{\nu} b^{1-\nu} \geq \frac{a+b-|a-b|}{2}.$$

When $n = 2$ and $\nu = \frac{1}{2}$, the inequality $a^{\frac{1}{2}} b^{\frac{1}{2}} \geq \frac{a+b-|a-b|}{2}$ is called the reverse Caucy inequality. A natural matrix form for two positive definite matrices A and B could be written as

$$A^{\frac{1}{2}}(A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^{\frac{1}{2}}A^{\frac{1}{2}} \geq \frac{A+B}{2} - \frac{|A-B|}{2}.$$

Furuichi, however, pointed out that the trace inequality

$$\mathrm{Tr}(A^{\frac{1}{2}}(A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^{\frac{1}{2}}A^{\frac{1}{2}}) \geq \frac{1}{2}\,\mathrm{Tr}(A+B-|A-B|)$$

is not true in general. That is,

$$\mathrm{Tr}(A\sharp B) \geq \mathrm{Tr}(A\nabla B - \frac{1}{2}|A-B|).$$

Recall that $A\sharp B = A^{\frac{1}{2}}(A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^{\frac{1}{2}}A^{\frac{1}{2}}$ is the geometric mean and $A\nabla_{\nu}B = \nu A + (1-\nu)B$ is the weighted arithmetic mean of A and B, respctively.

In this talk we introduce the operator inequalities for operator means such that

$$\phi(A)\sigma\phi(B) \geq \phi(A\nabla_{\nu}B) - \phi(r|A-B|)\ (r \geq 0)$$

for a non-negative operator convex function ϕ on $[0, \infty)$ and all positive definite matrices A and B, and we show the characterization of σ.

Abstracts of KOTAC
Volume 15(2016), 14–14

Reproducing kernel Hilbert space *vs.* frame estimates

Myung-Sin Song
Southern Illinois University, U.S.A.
myungsin.song@gmail.com

We consider conditions on a given system $\mathcal{F}$ of vectors in Hilbert space $\mathcal{H}$, forming a frame, which turn $\mathcal{H}$ into a reproducing kernel Hilbert space. It is assumed that the vectors in $\mathcal{F}$ are functions on some set Ω. We then identify conditions on these functions which automatically give $\mathcal{H}$ the structure of a reproducing kernel Hilbert space of functions on Ω. We further give an explicit formula for the kernel, and for the corresponding isometric isomorphism. Applications are given to Hilbert spaces associated to families of Gaussian processes.

Abstracts of KOTAC
Volume 15(2016), 15–15

Preservers of completely positive matrix rank

LeRoy B. Beasley
Utah State University, U.S.A.
leroy.b.beasley@aggiemail.usu.edu

Let $\mathcal{M}_{m,n}(\mathbb{R})$ denote the set of $m \times n$ matrices with entries in $\mathbb{R}$. We write $\mathcal{M}_{m,n}(\mathbb{R}_+)$ to denote the subset of $\mathcal{M}_{m,n}(\mathbb{R})$, all of whose entries are nonnegative. Let $\mathcal{S}_n(\mathbb{R})$ denote the set of all $n \times n$ real *symmetric* matrices. A matrix $A \in \mathcal{S}_n(\mathbb{R})$ is said to be *completely positive* if there is some matrix $B \in \mathcal{M}_{n,k}(\mathbb{R}_+)$ such that $A = BB^t$. The *CP-rank* of the matrix A is the smallest k such that $A = BB^t$ for some $B \in \mathcal{M}_{n,k}(\mathbb{R}_+)$. In this talk we investigate the linear operators on $\mathcal{S}_n(\mathbb{R})$ that preserve sets of matrices defined by the CP-rank. We classify those that preserve the CP-rank function, those that preserve the set of CP-rank-1 matrices, those that preserve the sets of CP-rank-1 matrices and the set of CP-rank-2 matrices, and those that strongly preserve the set of CP-rank-1 matrices.

Abstracts of KOTAC
Volume 15(2016), 16–16

Extensions of Buzano inequality

Masatoshi Fujii
Osaka kyoiku University, Japan
mfujii@@cc.osaka-kyoiku.ac.jp

Buzano inequality is an interesting extension of Schwarz inequality.

Buzano inequality (BI) Let H be an inner product space, and $a,\ b,\ x \in H$. Then

$$|(a,x)(x,b)| \leq \frac{1}{2}(\|a\|\|b\| + |(a,b)|)\|x\|.$$

In this talk, we discuss some extensions of (BI). One is a simultaneous extension of Diaz-Metcalf inequality and (BI). The other is an inner product C^*-module version of (BI), in which Schwarz inequality is expressed in terms of the operator geometric mean # as

$$|(x,y)| \leq u^*(x,x)u \ \# \ (y,y),$$

where u is the partial isometry in the polar decomposition of (x,y).

Abstracts of KOTAC
Volume 15(2016), 17–17

Wavelets and spectral triples for fractal representations of Cuntz algebras

Sooran Kang
University of Colorado, U.S.A.
soorankang@gmail.com

The goal of this talk is to describe the connections between graph C^*-algebras, wavelets on fractals and spectral triples. In particular we discuss the relation between the wavelet decompositions of certain fractal representations of graph algebras and the eigenspaces of Laplacians associated to spectral triples on the ultrametric Cantor sets of graphs. For simplicity, we only discuss the case that the vertex matrices of finite directed graphs consist of $\{0, 1\}$ entries and the associated representations give Cuntz algebras in this talk. This is a joint work with Carla Farsi, Elizabeth Gillaspy, Antoine Julien and Judith Packer.

Abstracts of KOTAC
Volume 15(2016), 18–18

α-Weyl operators

Slaviša V. Djordjević
Benemerita Universidad Autonoma de Puebla, Mexico
slavdj@fcfm.buap.mx

Let H be a (complex) Hilbert space and $h = \dim(H)$ denote the Hilbert dimension of the space H, where $h > \aleph_0$. Then any nonzero proper closed two-side ideal in $B(H)$ is the form $\{K \in B(H) : \dim \overline{R(K)} < \alpha\}$, for some $0 \leq \alpha < h$. Denote this ideal with $\mathcal{F}_\alpha(H)$. We denote the kernel of an operator by $N(T)$, $n(T) = \dim N(T)$, and $n^*(T) = \mathrm{codim}R(T)(= \dim H/R(T))$, where $R(T)$ denote the rank of an operator. Using classical (Atkinson) characterization of Fredholm operators, we can define α-Fredholm operators like those that are invertible modulo $\mathcal{F}_\alpha(H)$.

The main purpose of the present talk is study the α-Weyl operators, compare different ways of their introduction and give some of properties.

Abstracts of KOTAC
Volume 15(2016), 19–19

Inner functions associated with block Hankel operators

Dong-O Kang
Chungnam National University, Korea
dokang@cnu.ac.kr

It is known that for a matrix-valued function $\Phi \in L^\infty_{M_n}$, the kernel of a block Hankel operator H_Φ has the form $\Theta H^2_{\mathbb{C}^m}$ for a natural number m and an $n \times m$ matrix inner function Θ. It will be shown that the size of the inner functions associated with the kernels of block Hankel operators is decided by a certain independency of the columns of the symbol functions.

Abstracts of KOTAC
Volume 15(2016), 20–20

A relation between weighted shifts on directed trees and 2-variable weighted shifts

Jasang Yoon
The University of Texas Rio Grande Valley, U.S.A.
jasang.yoon@utrgv.edu

In this talk, we define 2-variable weighted shifts on directed trees and introduce some relations between weighted shifts on directed trees and 2- variable weighted shifts for hyponormality, subnormality, and quasinormality.

Abstracts of KOTAC
Volume 15(2016), 21–21

Residual quotients and the Taylor spectrum

Robin Harte
Trinity College, Ireland
rharte@maths.tcd.ie

An old idea of "residual quotient" furnishes the platform from which the proof of the several variable spectral mapping theorem for the left spectrum is launched. Something more complicated is needed for the Taylor spectrum; without commutivity, even the one way spectral mapping theorem fails here.