Weighted shifts on directed trees: a subnormal completion problem and some related properties

Il Bong Jung

Kyungpook National University, Daegu, 41566 Korea
E-mail: ibjung@knu.ac.kr

Given a directed tree and a collection of weights on a subtree, the subnormal completion problem is to determine whether the weights may be completed to the weights of an injective, bounded, subnormal weighted shift on the Hilbert space arising from the full tree. We study this problem for weighted shifts S_λ on directed trees $\mathcal{T}_{\eta,\kappa}$, where $\mathcal{T}_{\eta,\kappa}$ is a directed tree with a single branching vertex, η branches and the trunk of length κ. We give several characterizations of when such a completion is possible. Also we discuss some explicit solutions for subnormal completion problem in a special low number generation. In addition, we discuss some operator properties of S_λ via classical weighted shifts, the so-called "the i-th branching weighted shifts" $W^{(i)}$ for $0 \le i \le \eta$, whose weights are derived from those of $S_{\boldsymbol{\lambda}}$.