

Finite operational partition of unity via conditional expectation

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A finite set $\{\lambda_i : 0 \leq \lambda_i \leq 1\}_{i=1}^n$ is called a finite partition of 1 if $\sum_i \lambda_i = 1$.

Starting from the notion of "finite partition of 1", we pick up an extended notion of "finite partition of 1" to the operator theory, which is the notion of "finite operational partition of unity (FOP for short)" introduced by Lindblad.

By using FOP of unity, we consider a notion of "operational convex combination" which is an extended version of usual "convex summation", and construct a "unital completely positive map (UCP)" from a given FOP of unity. In a special case, we arrive at the "conditional expectation".

As the next stage, by starting from the "conditional expectation" $E : M \rightarrow N$ for a given inclusion $N \subset M$ of finite factors, we consider some special FOP of unity arising from this conditional expectation E and discuss on a special UCP map Φ arising from this FOP of unity.