An inequality satisfied by m-expansive pairs of operators and its consequnces

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Abstract

A pair (A,B) of Banach operators $A,B\in B(\mathcal{X})$ is (m,P)-expansive, for some operator $P\in B(\mathcal{X})$ and positive integer m, if $\triangle^m_{A,B}(P)=(I-L_AR_B)^m(P)=\sum_{j=0}^m(-1)^j\begin{pmatrix} m \\ j\end{pmatrix}A^jPB^j\le 0$, whre L_A and R_B denote respectively the operators $L_A(X)=AX$ and $R_B(X)=XB$. Assuming that the pair (A,B) satisfies a positivity property (of type $L_AR_B\triangle^t_{A,B}(P)\ge 0$ whenever $\triangle^t_{A,B}(P)\ge 0$, t some non-negative integer), (m,P)-expansive pairs (A,B) satisfy

$$(L_AR_B)^n(P)\le\begin{pmatrix} n \\ m-1\end{pmatrix}\triangle^{m-1}_{A,B}(P)+\sum_{j=0}^{m-2}\begin{pmatrix} t \\ j\end{pmatrix}\triangle^j_{A,B}(P)$$

for all positive operators $P\in B(\mathcal{X})$ and integers $n\ge m$. We look at some consequences of this property and prove amongst other results that: (i) if T is (m,P)-expansive (resp., contractive) for some even (resp., odd) positive integer m, then T is $(m-1,P)$-expansive (resp., contractive); (ii) if (T^*,T), T and $P\in B(\mathcal{H})$, is (m,P)-symmetric for some positive even integer m and injective $P\ge 0$, then T is $(m-1,P)$-symmetric; (iii) invertible $T\in B(\mathcal{H})$ such that (T^*,T) is m-expansive are generalised scalar.

Most of the material of this talk is taken from the joint work of the presenter with Prof. I.H. Kim, amongst them the first five references below.

1. Structure of n-quasi left m-invertible and related classes of operators, Demonstratio Math. 53(2020), 249-268.

2. Structure of iso-symmetric operators, Axiom 2021,10(4),256, https://doi.org/10.3390/axioms10040256.

3. Expansive operators and Drazin invertibility, Rendi. Circ. Mat. Palermo (https://doi.org/10.1007/s12215-021-00683x)

4. Left m-invertibility by the adjoint of Drazin inverse and m-self-adjointness of Hilbert spaces, Lin. Multilin. Alg. (doi.org/10.1080/03081087.2020.793880)

5. Expansive operators which are power bounded or algebraic, Operators and Matrices 16(1)(2022).

6. B.P. Duggal, On (m,P)-expansive operators: products, perturbation by nilpotents, Drazin invertibility, Concrete Operators 2021;8:158-173.